A METHOD OF EVALUATING GRADINGS ON CONCRETE AGGREGATES

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by

S. POPOVICS

Joint Highway Research Project

PURDUE UNIVERSITY
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A METHOD OF EVALUATING GRADINGS OF CONCRETE AGGREGATES

TO: K. B. Woods, Director
Joint Highway Research Project

April 6, 1961

FROM: H. L. Michael, Assistant Director
Joint Highway Research Project

File: 5-9-7
Project: C-36-42C

Attached is a final report titled, "A Method of Evaluating Gradings of Concrete Aggregates" which has been authored by Sandor Popovics, formerly a graduate assistant on our staff. The research performed for this study, which was also used by Mr. Popovics as his Ph.D. thesis, was largely performed in Project Laboratories under the direction of Dr. W. L. Dolch of our staff.

The research here reported is concerned with the numerical characterization of an aggregate grading by means of the fineness modulus and the specific surface area concepts. Considerable theoretical and experimental work is reported and substantiation is obtained that in evaluating a grading the values of maximum particle size, fineness modulus and specific surface area should be employed.

The report is presented to the Board for the record.

Respectfully submitted,

Harold L. Michael
Secretary

HLM:kmc

Attachment

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ABSTRACT

This study is concerned with the numerical characterization of the grading of concrete aggregates.

The fineness modulus and specific surface area concepts are discussed. It is shown analytically that the fineness modulus is an average - specifically, it is proportional to the average of the logarithmic particle size distribution. It is also shown analytically that the specific surface area is a variance - specifically, it is proportional to the second moment of a particle size distribution function.

Since a distribution is characterized fairly well by its average and its variance, the hypothesis is made that an aggregate grading is adequately defined if its maximum size, fineness modulus, and specific surface area are given. A corollary of this hypothesis is that concrete with the same properties in both the plastic and hardened state will be produced by aggregate gradings that have the same numerical values of maximum size, fineness modulus, and specific surface area, regardless of the details of the gradings, and if other proportions of the mix are the same.

This proposition was tested. It is shown that certain results from the literature substantiate the hypothesis, particularly for the properties of hardened concrete.

In order to test the hypothesis more adequately a series of concrete mixes was designed to incorporate three gradings - a continuous, a one-gap, and a two-gap - that have the same values of the three parameters in question. These gradings were incorporated in mixes that were both rich, and

lean, and wet and dry. The concretes were made and the properties of slump, flow, bleeding capacity, segregation, unit weight, and compressive strength were determined, using standard procedures whenever possible. The experimental results were analyzed statistically, and the results of this analysis confirm the hypothesis to a reasonable degree of confidence.

A method is then presented for the use of this concept in combining various gradings to obtain desired values of fineness modulus and specific surface.

LIST OF SYMBOLS

d	particle size of aggregate,
d_{min}	minimum particle size,
D	maximum particle size,
d_a	average particle size,
$f(d)$	cumulative distribution function of particle size which gives the relation between the particle size d and the quantity of those particles that are smaller than d, per cent by weight,
p	proportion of each individual fraction to the whole aggregate,
n	number of fractions in an aggregate,
m	fineness modulus of aggregate grading,
m'	fineness modulus of the mixture of aggregate and cement (total solid volume),
m_o'	optimum fineness modulus of the grading of total solid volume,
F	first grading area,
s	hypothetical specific surface area of aggregate,
s_o	optimum value of the specific surface area,
S	second grading area,
γ	bulk specific gravity,
T	total of the i^{th} column.

INTRODUCTION

The proper grading of the aggregate component is generally considered to have an important effect on the quality of portland-cement concrete. This effect is sometimes minimized in practice by, for instance, the addition of extra cement, but such solutions to the problem are expensive and may be technically unsound.

This study deals with the numerical characterization of the aggregate grading by means of the fineness modulus and the specific surface area concepts. The greatest importance of the numerical characterization of a grading is that the grading is expressed in the same way as the cement content, water-cement ratio, strength, consistency, etc. Therefore, some relationships can be established among the grading of the aggregate and other properties of freshly-mixed or hardened concrete. This has been attempted by many researchers. Derivation of such relationships was also the subject of the writer's first thesis (1).[1]

Many methods of evaluating aggregate gradings have been proposed but all to date have been only partially suc-

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[1]Numbers in parentheses refer to the bibliography which is found at the end of the thesis.

cessful. For instance, if a grading is evaluated by specifying certain sieve-curves, many gap gradings that are really useful for concrete production will be rated as poor. The sieve-curve method, in other words, is too conservative. On the other hand if the fineness modulus is used to evaluate the gradings, many that are really unsuitable will be rated as good. That is, the fineness modulus method is too liberal.

The writer believes that the method using the maximum particle-size, fineness modulus, and specific surface area simultaneously for evaluating the aggregate grading is as suitable for gap as for continuous gradings, and that the precision of this method is not less than that of the usual method using sieve border curves for continuous gradings.

The term "optimum grading" may be defined as follows. Generally, the grading is considered technically optimum when one or more of the desired properties of the concrete in question is best assured (compressive strength, modulus of rupture, water permeability, density, durability, abrasion resistance, etc.), and when the maximum particle size, particle shape, texture, cement content, water-content, method of compaction, etc., are specified. Thus, the optimum of grading is partly a function of concrete composition, and partly a function of the requirements demanded of the concrete. Since existing tests in connection with fineness modulus are confined to the relationship between fineness

modulus and compressive strength, the present study also generally uses the expression "optimum" in referring to the compressive strength. However, the writer has the opinion that similar optimum of gradings, i.e., the optimum values of fineness modulus and specific surface area, could be established also for other concrete properties.

In the study the effects of particle shape, texture, etc., are not considered but rather it is assumed that these effects are the same in every case of grading. This assumption is not true, but is acceptable because these effects influence only the optimum values of the fineness modulus and the specific surface area. The method using fineness modulus and specific surface area for evaluation is itself independent of these assumptions, and the subject of this study is restricted to a discussion of the evaluating methods.

The previous statement referring to the particle shape includes the assumption that particle shape is sherical and the surface texture of particles is smooth. In this way not the true value of the specific surface area is calculated but a hypothetical one that is always smaller than the true one.

The evaluation of grading that takes into consideration both the fineness modulus and the specific surface area is a little more complicated than, for instance, the use of the fineness modulus alone. Therefore it could be employed mainly by ready-mix concrete plants and prefabrication factories

or in areas where only gap grading can be produced economically from the locally-available aggregates.

In proof of the fundamental principles, some statements of mathematical analysis and statistics will be used. The mathematical definition of the average, the second moment, and the variance of a statistical distribution are needed (2):

$$\mu = \text{ave } (x) = \int_a^b x.f'(x)dx \qquad \text{A.)}$$

$$\mu_2 = \text{ave } (x^2) = \int_a^b x^2.f'(x)dx \qquad \text{B.)}$$

$$\sigma^2 = \text{var } (x) = \text{ave}(x-\mu)^2 = \int_a^b (x-\mu)^2.f'(x)dx = \mu_2 - \mu^2 \qquad \text{C.)}$$

where

$f(x)$	cumulative distribution function of the variable x,
$f'(x)$	first derivative of this function,
μ	mean or average value of x,
μ_2	second moment of x, and
σ^2	variance of x.

THE THEORETICAL BASIS OF THE FINENESS MODULUS CONCEPT

Some Properties of the Fineness Modulus

The fineness modulus was presented by Abrams on the basis of his researches about 40 years ago to characterize the grading of concrete aggregates by a single number (3). In the period that has passed since then, there has been much discussion of the fineness modulus. Therefore, at present the advantages and disadvantages of the fineness modulus are distinct enough. The fineness modulus concept is simple and is applicable over a large range. However, its utility is limited.

Advantages of the Fineness Modulus

In general, the fineness modulus characterizes the grading of aggregates satisfactorily, with respect to concrete technology. This means, according to Abrams, that any other grading having the same fineness modulus "will require the same quantity of water to produce mix of the same plasticity and gives concrete of the same strength, so long as it is not too coarse for the quantity of cement used" (3).

The fineness modulus characterizes the grading by one number. The fineness modulus method is more general than

other methods, because by means of it not only continuous gradings, but also gap gradings can be evaluated under certain conditions. Other methods cannot do this as simply.

The fineness modulus is more adaptable than the other methods. Knowing the optimum values of the fineness modulus, one can take into account the effects of maximum size, cement content, and particle shape for the determination of the optimum grading.

The blending of two or more aggregate components, which is frequently necessary to meet specifications, can easily be computed.

Disadvantages of the Fineness Modulus

The fineness modulus theory has only two important disadvantages, a theoretical one and a practical one. The objection is often raised that the fineness modulus lacks a theoretical base and that it is entirely empirical and arbitrary. So, according to ASTM standard (4), the fineness modulus is "an empirical factor." Or, for example (5), the fineness modulus is "an empirical factor, an arbitrary function of grading of the aggregate." Duriez (6) writes about the fineness modulus (translation): "Unfortunately, this method does not have any rational basis and it is entirely empirical."

There are gradings, mainly gap gradings, for which the numerical value of the fineness modulus is proper although the grading is not satisfactory for use in concrete. For example,

if D = 1½", various gradings of fineness modulus of m = 5.5 can be produced so that the fine sand[2] content varies between 0 and 25 per cent. However, if the actual fine sand content is near the lower limit, the workability and density of concrete made with such grading cannot be satisfactory because of the lack of fine particles. The fine sand content near the upper limit is not satisfactory either because it is too high and so the water need of aggregate is also too high for practical purposes. So, the limits of utility of the fineness modulus concept are uncertain, and one reason for this is the lack of a theoretical basis.

Some Complementary Comments

One purpose of this section is to present a theoretical explanation of the fineness modulus concept or, in other words, to show what this "empirical factor" really is. On the basis of this, it can be seen better under what circumstances the fineness modulus can be used in a satisfactory manner.

In general, the particle size of the aggregate will be expressed in millimeters (mm) in this paper considering that the standardized sizes of the sieve openings are given also in millimeters. Sometimes, however, inches are used, e.g., when it is expedient.

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[2] fine sand: particles, passing sieve #16.

The Fineness Modulus According to Abrams

The Fineness Modulus as a Sum of Ordinates

The definition of the fineness modulus according to Abrams agrees essentially with the present ASTM standard (4). Accordingly, the fineness modulus is "obtained by adding the total percentages of a sample of the aggregate retained on each of the specified series of sieves, and dividing the sum by 100.

Note: The sieve sizes used are No. 100 (149-micron), No. 50 (297-micron), No. 30 (590-micron), No. 16 (1190-micron), No. 8 (2380-micron), and No. 4 (4760-micron) and 3/8 in., 3/4 in., 1½ in., and larger, increasing in the ratio to 2 to 1."

The mathematical form of this definition is:

$$m = \frac{\sum_{i=1}^{n} b_i}{100} \qquad 1.)$$

where

b_i total percentages of a sample of the aggregate retained on the i^{th} member of the specific series of sieves (Figure 1).

In accordance with the above definition one can also speak about the fineness modulus of the mixture of aggregate and cement.

The Fineness Modulus Measured by an Area

Even Abrams had pointed out (3) that the fineness modu-

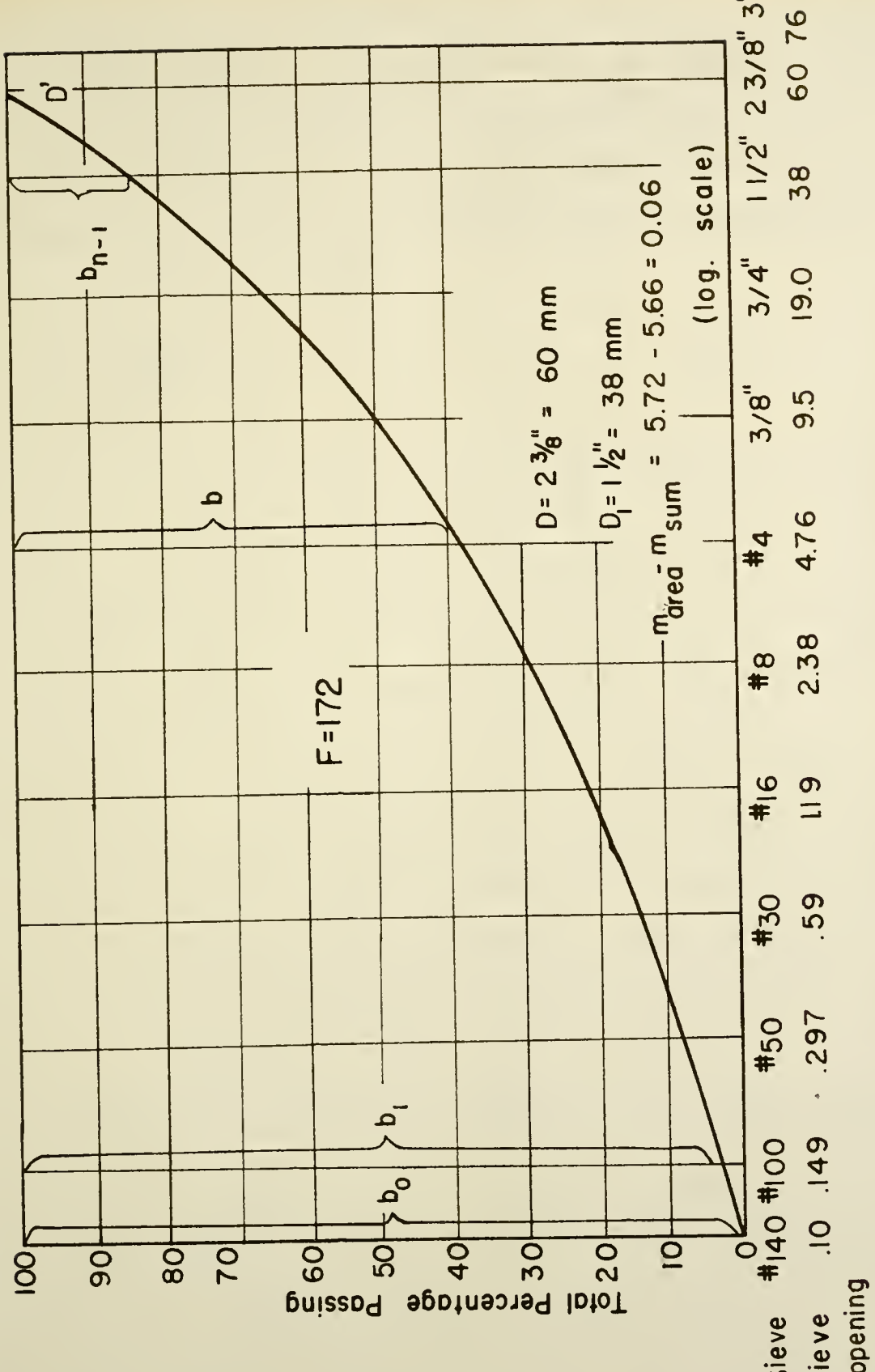

Figure 1. Determination of the Fineness Modulus

lus is proportional to the area F which is surrounded by the grading curve and the axis of ordinates in the system of co-ordinates of Figure 1. This area F can be called the "first grading area." From the value of the area F, the fineness modulus can be calculated:

$$m = \frac{F}{30.1} \qquad 2.)$$

Proof:

The surface of F first grading area can be calculated on the basis of Figure 1. This is nothing else but the sum of the surfaces of trapesoids which are indicated by the intersections of the grading curve and ordinates belonging to the respective members of the specified series of sieves.

Abrams did not fix the position of the axis of ordinates exactly in respect to area F. On the basis of Hummal's proposal (7), the European technical literature places the origin of the system of co-ordinates at the point of d = 0.1 mm. In this case:

$$F = \frac{b_0+b_1}{2} \log 1.5 + \frac{b_1+b_2}{2} \log 2 + \frac{b_2+b_3}{2} \log 2 + \ldots + \frac{b_{n-1}}{2} \log \frac{D}{D_1} \quad (*)$$

A consideration of formula 1.), indicates that equation 2.) follows at once from equation (*):

$$\frac{F}{\log 2} \cong b_1 + b_2 + \ldots + b_{n-1} = 100\ m$$

where

$b_1, .. b_{n-1}$ total percentages of a sample of the aggregate retained on each of the specified series of sieves, and

D_1 nearest sieve size of the specified series of sieves below the value D.

The proof points out that the relationship 2.), between m and F is really only approximate. However, the value of difference between the fineness modulus calculated from formula 1.) (m_{sum}) and the one calculated from formula 2.) (m_{area}) can be determined from equation (*). Using the symbols of the above proof, this is the following (Figure 1):

$$100\,(m_{area}-m_{sum})=0.29b_0-0.21b_1-0.5b_{n-1}\,(1-3.32\log\frac{D}{D_1}) \quad 3.)$$

If the D maximum size coincides with the measurements of some sieve of the specified series (i.e., $D=D_1$), then the last part of the right side of equation 3.), which contains the term b_{n-1}, disappears. Equation 3.) shows that the difference between the fineness moduli which are calculated in two different ways is less than 0.1 in most cases. Thus, the approach of formula 2.) is entirely satisfactory for practical use.

The Fineness Modulus as a Logarithmic Function of Particle-Size

The following relationship between the fineness modulus and particle size comes from Abrams (3):

$$m = 7.94 + 3.32 \log d" \qquad 4.)$$

About equation 4.) Abrams says: "The relation applies to a single size material or to a given particle. The fineness modulus is then a logarithmic function of the particle."

The Fineness Modulus Calculated from Fractions

If the aggregate is separated in various fractions by sieves, then, a knowledge of the fineness moduli and the proportions of each fraction makes possible the calculation of the fineness modulus of the entire aggregate as follows:

$$m = \sum_{i=1}^{n} \frac{m_i p_i}{100} \qquad 5.)$$

Two New Forms of the Fineness Modulus

The Analytical Form

Previously it was stated that the determination of F first grading area can be obtained by summarizing the following terms:

$$\frac{b_i + b_{i+1}}{2} \log \frac{d_i}{d_{i+1}}$$

This is numerical integration. If the function of the grading curve is known, then, there is no difficulty in performing the analytical integration. The following form is obtained (8):

$$m = \frac{F}{30.1} = 0.0332 \left[100 \log (10D) - 0.4343 \int_{0.1}^{D} \frac{f(d)}{d} \, dd \right] \quad 6.)$$

The base of the logarithm is 10.

Proof:

The area F can be expressed (as a Stieltjes integral) in the following way, (Figure 1):

$$F = \text{area } (0.1\text{-}D\text{-}D'\text{-}100) - \text{area } (0.1\text{-}D\text{-}D') =$$

$$= 100 \log (10D) - \int_{0.1}^{D} f(d) \, d\log (10d) \quad (*)$$

But the following equality exists (9):

$$\int_{0.1}^{D} f(d) d\log(10d) = 0.4343 \int_{0.1}^{D} \frac{f(d)}{d} \, dd$$

By substituting this in the equation (*), formula 6.) is obtained.

The Fineness Modulus as an Average Concept

The fineness modulus can be expressed also in the language of statistics (10), (11):

$$30.1m = \int_{0.1}^{D} \log (10d) f'(d) dd = \text{ave} \left(\log(10d) \right) \quad 7.)$$

Proof:

The first equality of formula 7.) follows from integration by parts of the right side of equation 6.). If this is compared with the formula A.) of the Introduction, the correctness of the second equality of equation 7.) will become obvious.

Relation 7.) shows quite strictly that the value of the fineness modulus is proportional to the logarithmic average of particle sizes of f(d) grading.[3]

If this average particle size expressed in mm is marked with d_a, then:

$$30.1m = 100 \log 10d_a \qquad 7.1.)$$

In other words, a larger value of fineness modulus means a bigger average particle size, i.e., a coarser grading.

The relation 7.) is an important statement because it is the general theoretical basis for the fineness modulus, a basis missing until now.

Here, however, the question is raised, why exactly the logarithmic average of all the various statistical averages is significant with respect to evaluation of grading. Why not some other average, e.g., the value of arithmetical mean? The answer is the following: on the basis of Kolmogorov's research (12) it is known that the gradings

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[3]This statement is really a generalization of Abrams' formula 4.). Its validity can be visualized--in addition to the above stated strict mathematical proof--by means of carrying out an area-equalization on the F grading area shown in Figure 1.

of crushed stones, and often also that of the natural rounded aggregates, follow a special statistical distribution, the so-called logarithmic normal distribution. The role of the arithmetic mean of a normal distribution is taken over in case of logarithmic normal distribution by the logarithmic average (13), i.e., by the fineness modulus.

Relying upon these findings, it is seen that the fineness modulus is not merely an empirical or arbitrary function of grading, but theoretically an important factor. From this statement, the answer to the following two questions can be found:

a) Why can the fineness modulus be used so satisfactorily to characterize many gradings?

b) Why does it not characterize all gradings, the extreme ones too?

The logarithmic normal distribution (like the common normal distribution) can be completely characterized by two parameters, namely by its average and variance. However, the fineness modulus method used only the average to evaluate the grading. Thus, this method cannot give correct, comparable results if the variance of the particle-size distribution is either extremely high or too small.

If, however, the value of the variance is not extreme--which is the case for continuous gradings used in practice--then the fineness modulus gives good characteristics of the grading.

On the basis of what has been said on the preceding page, the essentials of the fineness modulus can be summarized as follows: The fineness modulus is a number which is proportional to the logarithmical average of particle size of grading; its value is illustrated by the F, first grading area, which is determined by the cumulative distribution function of grading. The calculation of a fineness modulus may be made by means of some suitable formula or by adding the total percentages of a sample of the aggregate retained on each of a specified series of sieves and dividing the sum by 100.

The fineness modulus of the mixture of cement and aggregate is proportional to the logarithmical average of particle size of the total solid volume.

Optimum Fineness Moduli and Optimum Average Particle-Sizes

This chapter presents an application of the previous theoretical results.

To use the fineness modulus for grading evaluation, one needs its optimum values. These are generally the maximum permissible values of the fineness modulus which are given as function of maximum particle size, cement content, and particle shape. If the fineness modulus of grading is lower than the optimum, the water requirement of concrete mix increases. On the other hand, if the fineness modulus is higher than the optimum, the workability of concrete decreases.

Abrams was the first to publish the maximum permissible

values of the fineness modulus (3). These values are shown in Figure 2. Following Abrams, many researchers dealt with this problem, among them Americans(14), (15), (16). However, these results did not show any essential difference from Abrams' results.

Several researchers also tested the optimum values of the fineness modulus for total solid volume, that is, for a mixture of cement and aggregate. Walker dealt with this problem first in the United States (17). The different results clearly show the fact "that the combined optimum fineness modulus for total solid volumes is essentially constant for a given set of aggregates regardless of the cement content of the mixes involved." (16). This statement is in accordance with the experience that within practical limits, the consistency of concrete remains nearly constant if the type and gradation of the aggregates and the water content per unit of fresh concrete remain constant--regardless of the richness of mix (18).

But this experimental fact can be expressed on the basis of formulas 7.) - 7.1.) as follows:

> In case of a given particle shape and maximum size, the necessary condition for workability of freshly mixed concrete is that the logarithmical average of particle size of total solid volumes should not exceed an upper limit.

The d_{ao} column of Table 1 lists the optimum values of the average particle sizes. These were calculated by formula

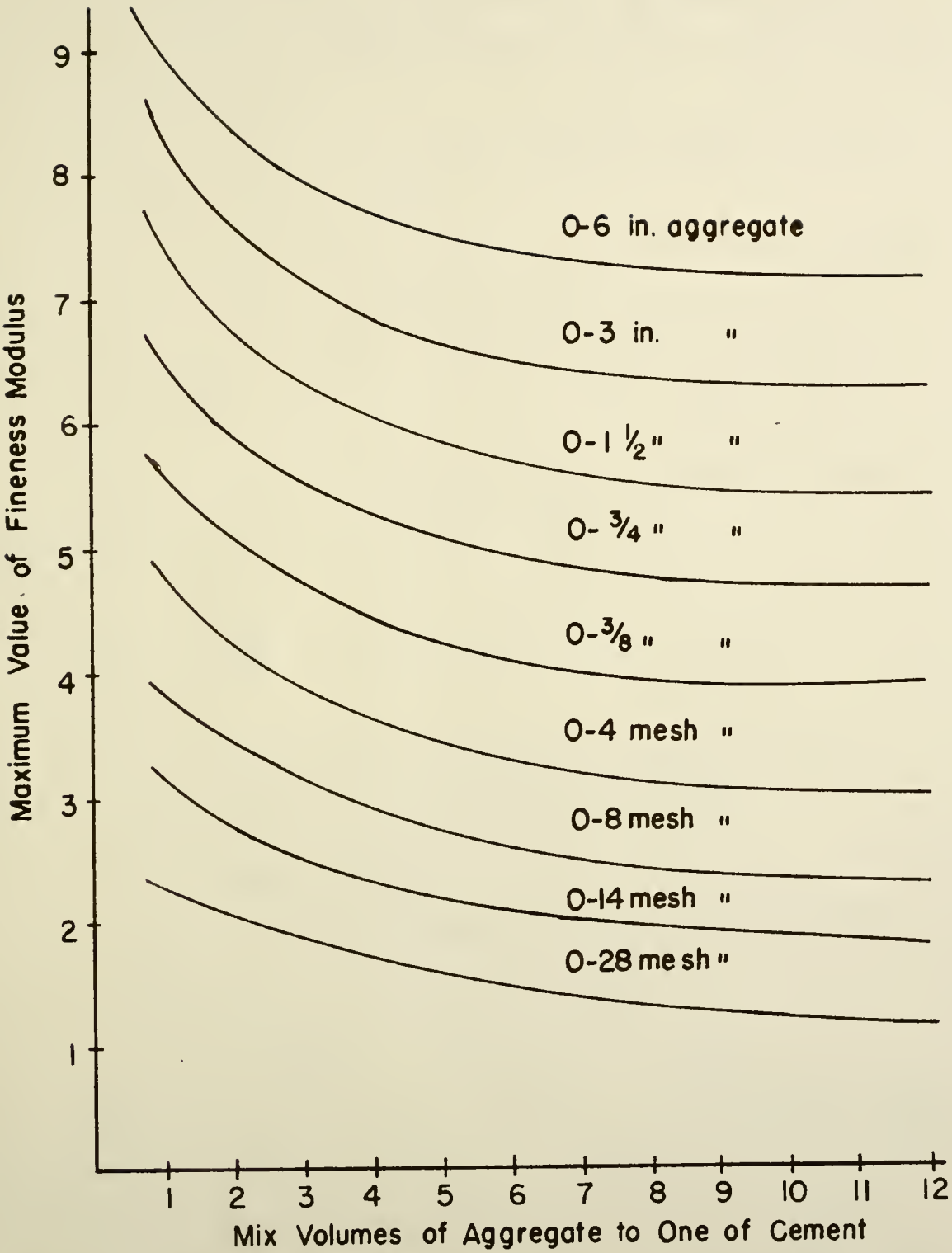

Figure 2. Maximum Permissible Values of Fineness Modulus of Aggregate. (Data from Abrams (3).)

7.1.) from the averages of the values of optimum fineness moduli published by Walker and Bartel for mixtures of cement, sand, and pebbles (19).

Figure 3 was prepared on the basis of Table 1. This figure illustrates two facts:

a. The approximate m_o' values of the combined optimum fineness moduli can be obtained within the limits of D = 1/10" - 6" (=2.4 - 152 mm.) from the following equation:

$$m_o' = 2.45 \log D'' + 4.5 \qquad 8.)$$

b. A good approximate relationship between D maximum particle size and the ratio $\frac{d_{ao}}{D}$ is the following:

$$\log \left(100 \frac{d''_{ao}}{D''}\right) = -0.263 \log D'' + 0.95 \qquad 9.)$$

Proof:

a. Equation 8.) was obtained from the data shown in Table 1 by the method of least squares.

b. From equation 8.):

$$30.1\, m = 100 \log 254\, d_{ao}'' = 73.7 \log D'' + 135.5$$

$$\log d''_{ao} = 0.737 \log D'' - 1.050$$

$$\log 100 \left(\frac{d''_{ao}}{D''}\right) = -0.263 \log D'' + 0.95$$

Equation 9.) shows as a function of the D maximum size, what fraction of the D maximum size the d_{ao} average particle size may have in order to insure an adequate workability.

Equation 9.) represents the relationship between grading and workability from a new point of view.

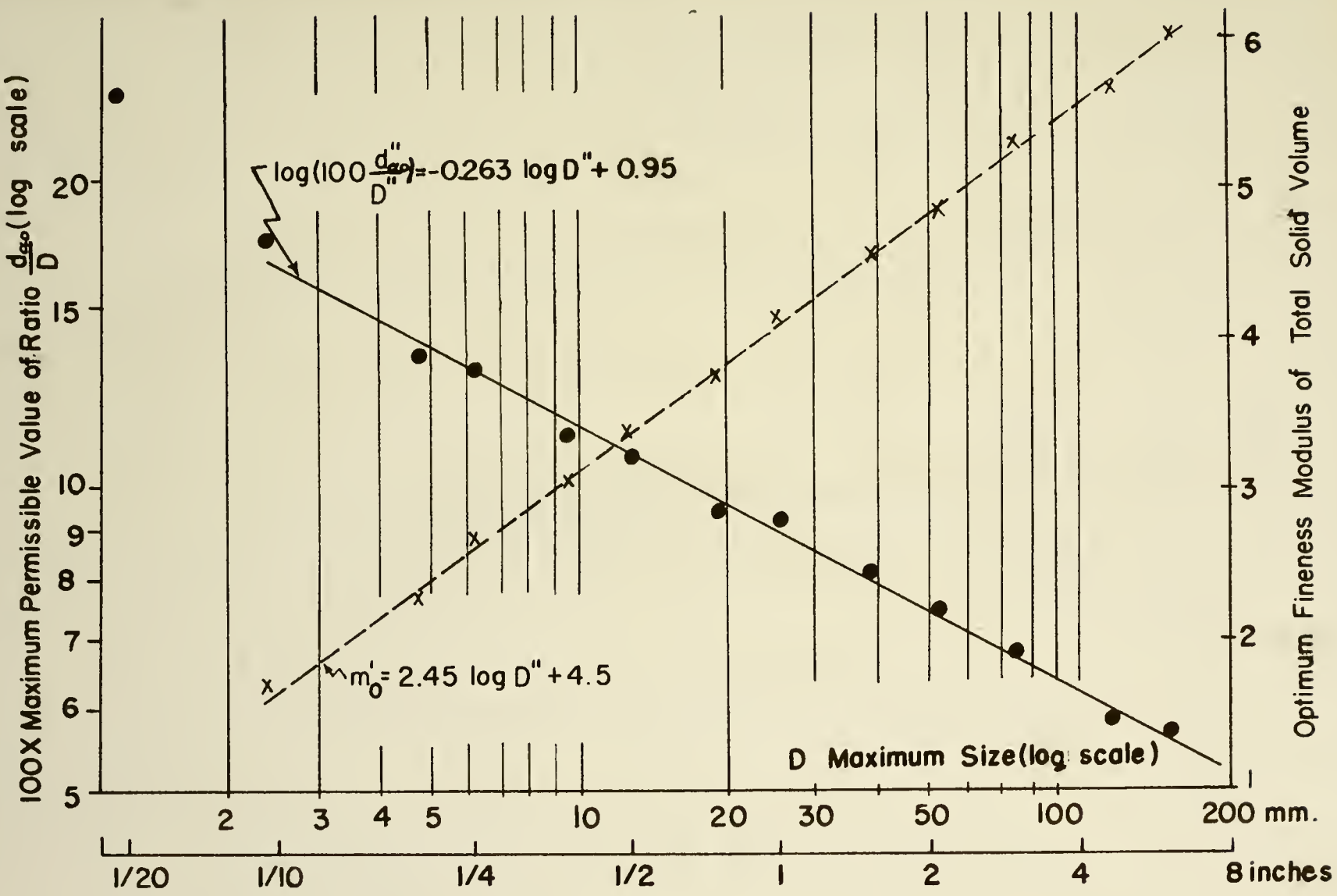

Figure 3. Maximum Permissible Values of Average Particle-size and of Fineness Modulus

TABLE 1

OPTIMUM AVERAGE PARTICLE SIZES OF TOTAL SOLID VOLUME OF DIFFERENT MAXIMUM SIZE

Size of Aggregate	Max. Size D in mm	Optimum Fineness Modulus of Total Solid Volume (19)	d_{ao} in mm	$\frac{d_{ao}}{D}$
0-No. 28	0.6	1.00	0.200	0.333
0-No. 14	1.2	1.54	0.290	0.242
0-No. 8	2.4	2.08	0.424	0.177
0-No. 4	4.8	2.68	0.642	0.133
0-No. 3	6.3	3.03	0.820	0.130
0-3/8"	9.5	3.42	1.07	0.113
0-1/2"	12.7	3.78	1.37	0.108
0-3/4"	19	4.17	1.80	0.095
0-1"	25.4	4.54	2.33	0.092
0-1-1/2"	38	4.93	3.04	0.080
0-2.1"	53	5.29	3.92	0.074
0-3"	76	5.70	5.21	0.0685
0-4-1/2"	115	6.07	6.73	0.0585
0-6"	152	6.45	8.75	0.0575

The conditions for the validity of equations 8.) and 9.) are the same as those of the fundamental values of m_o'.

THE SPECIFIC SURFACE AREA AS A STATISTICAL CONCEPT

One of the oldest ideas in concrete technology is the use of the numerical value of the specific surface area of the aggregate particles to evaluate a grading. The basis of this idea was the requirement that the surface of the particles should be entirely covered with cement paste. Consequently, the numerical value of the specific surface area of the aggregate should be related to the quantity of the two most important concrete components, namely cement and water, and be independent of the details of grading - at least within some limits.

Edwards (20) and Young (21) were the first to summarize this idea.

After the statistical interpretation of the fineness modulus concept, perhaps it will be interesting to test the specific surface area concept in the same regard.

The Specific Surface Area as a Variance Concept

Besides its geometric sense, the specific surface area also has a statistical interpretation. The surface area of an aggregate is characteristic of the variance of the particle-size distribution. This statement is more exactly expressed in the following way.

The specific surface area of an aggregate is proportional

to the second moment of a distribution function of particle size and, therefore, this can be a characteristic value of the variance of the distribution.[4] The specific surface area is given by the following formula:

$$s = \beta \int_{d_{min}}^{D} \frac{f'(d)}{d} dd \qquad 10.)$$

where

β constant which depends on the shape of particles, units, etc.

Proof:

a) The surface area of some particle, supposedly smooth, is the following:

$s' = \alpha d^2$

where

s' surface area of a particle,

d dimension of the particle

α a constant that depends on the shape of the particle, units, etc.

If there are several groups that consist of a different d_i particle size, and n_i particle number, then the sum of the surface areas is the following:

---

[4]Simultaneously, the second moment is also an average concept. Namely, it is the average of the values of x^2. In this particular case it is also the reciprocal harmonic average of the f(d) function. This thesis utilizes only the variance property of the specific surface area concept. These statements are valid for any particle shape if this shape is not a function of d.

$$s = \sum_{i=1}^{n} n_i s'_i = \alpha \sum_{i=1}^{n} d_i^2 n_i \qquad (*)$$

Convert from the sum of surface area to the specific surface area. In addition, assume that the grading is delineated by a h(d) cumulative grading function that is considered as differentiable. This h(d) gives the relation between the particle size d and the relative number of pieces that are smaller than d. This means that, in place of n_i in the equation (*), the value h'(d) dd should be substituted and the summation becomes the integration. Consequently, the equation (*) will now become the following:

$$s = \alpha \int_{d_{min}}^{D} d^2 h'(d) dd \qquad (**)$$

A comparison of equation (**) with equation B.) of the Introduction, shows that the specific surface area is indeed the second moment of h(d) where the particle size distribution function is taken by the number of particles. Thus, according to equation C.) of the Introduction, this is also a characteristic value of the variance of the distribution. In concrete technology it is not usual to give the grading in terms of the numbers of pieces; the method that is generally used is based on the relative weights of fractions. Therefore, equation (**) should be transformed. Let the f(d) function show the particle size

distribution taken by the relative weights. The weight can be indicated by the third power of d, because the weight of a spherical particle is:

$$w = \gamma_f \frac{\pi}{6} d^3$$

Therefore:

$$\beta f'(d) = \alpha d^3 h'(d)$$

Substituting into equation (**) gives equation 10.) as the final result.

Some Methods of Calculating the Specific Surface Area

In the following calculations the assumptions are always made that the aggregate particles are spherical and that the surface texture is smooth.[5]

The specific surface area calculated in this way is characteristic of the grading, but it is not the true surface area. The smoother and more spherical the particles, the better the agreement between the values of the hypothetical specific surface area and those of the true specific

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[5] If this simplifying assumption were not made, then it would be necessary to assume other particle shape or shapes in order to calculate the specific surface area and this would also be an approach. There exist various methods of measuring the specific surface area which work without any assumption regarding the particle shape. These methods do not give the actual value of the specific surface area but only quantities which are more or less proportional to it, and the exact value of the proportional factor is not known. In addition, if the specific surface area of the same aggregate is determined by other methods, different values result because the various methods express the various properties of the surface area in different ways.

surface area.

The calculation of specific surface area of a "one size" aggregate is simple and well known. The calculation of specific surface area of an aggregate fraction can be performed on the basis of its average particle size (20) (22), or by means of more accurate formulas (10).

A knowledge of the specific surface area of each individual fraction (s_i) allows the specific surface area of the entire aggregate to be calculated in the following way:

$$s = \sum_{i=1}^{n} \frac{s_i p_i}{100} \qquad 11.)$$

Two more general methods will be presented below.

The Integral Method

If the function of grading is known and it is differentiable, the specific surface area can be calculated in one step by means of formula 10.). The values of β can easily be determined for the various units of measure. If d is expressed in mm, and s in m^2/kg, then

$$s = \frac{6}{100\gamma} \int_{d_{min}}^{D} \frac{f'(d)}{d}\, dd \qquad 10.1.)$$

The Specific Surface Area Measured by an Area

The specific surface area can be determined on the basis of the following statement: The specific surface area is proportional to an area S that is bounded by the

absciss axis and the differential curve f'(d) in the semilogarithimical system of co-ordinates (Fig. 4.). This area can be called the "second grading area." The specific surface area can be calculated from the area S:

$$s = \frac{13.82}{100\gamma} S \qquad 12.)$$

where

S area under the differential curve in a semilogarithimical system of co-ordinates if d is expressed in mm.

Proof:

The following equality exists (9):

$$\int_{d_{min}}^{D} \frac{f'(d)}{d}\, dd = 2.30 \int_{d_{min}}^{D} f'(d)\, d\log d$$

However, the integral of the right side of this equation (a Stieltjes integral) represents the area S under the differential curve between the limits d_{min} and D in the semilogarithimical system of co-ordinates (Fig. 4).

Figure 4 gives an example for the $y' = f'(d)$ curve and for determining the area S. The ordinates of this chart were calculated from the following formula:

$$y_{i+1,i} = \frac{y_{i+1} - y_i}{d_{i+1} - d_i} \qquad 13.)$$

E.g., the ordinate belonging to the diameter-section of 0.4×10^{-2} and 0.2×10^{-2} mm is, as follows:

$$y(.004,.002) = \frac{0.02 - 0}{(0.4-0.2)\ 10^{-2}} = 10.0$$

This area method has some advantages. First of all, it is visual. In addition, it does not require a knowledge of the grading function. The area S can be directly determined from the results of a sieve test.

The precision of the numerical value of the area S can be increased slightly by means of one of Sheppard's corrections for grouping (23). Its purpose is to remove bias from the second moment that is calculated from grouped data instead of from the individual values.

Two Further Notes

In calculating the specific surface area, two factors were neglected. First, the surface area of very small particles not with full value was taken into account. The second omission came from the assumption that the shape of particles was spherical or cubic.

Omission of the Very Small Particles

The specific surface area of the very small particles should be partially neglected because their actual water requirement is less than that which would follow from their surface area. Kennedy (24) says: "Why shouldn't it be neglected? The cement paste consists of discrete particles surrounded by water; it is impossible to conceive of this

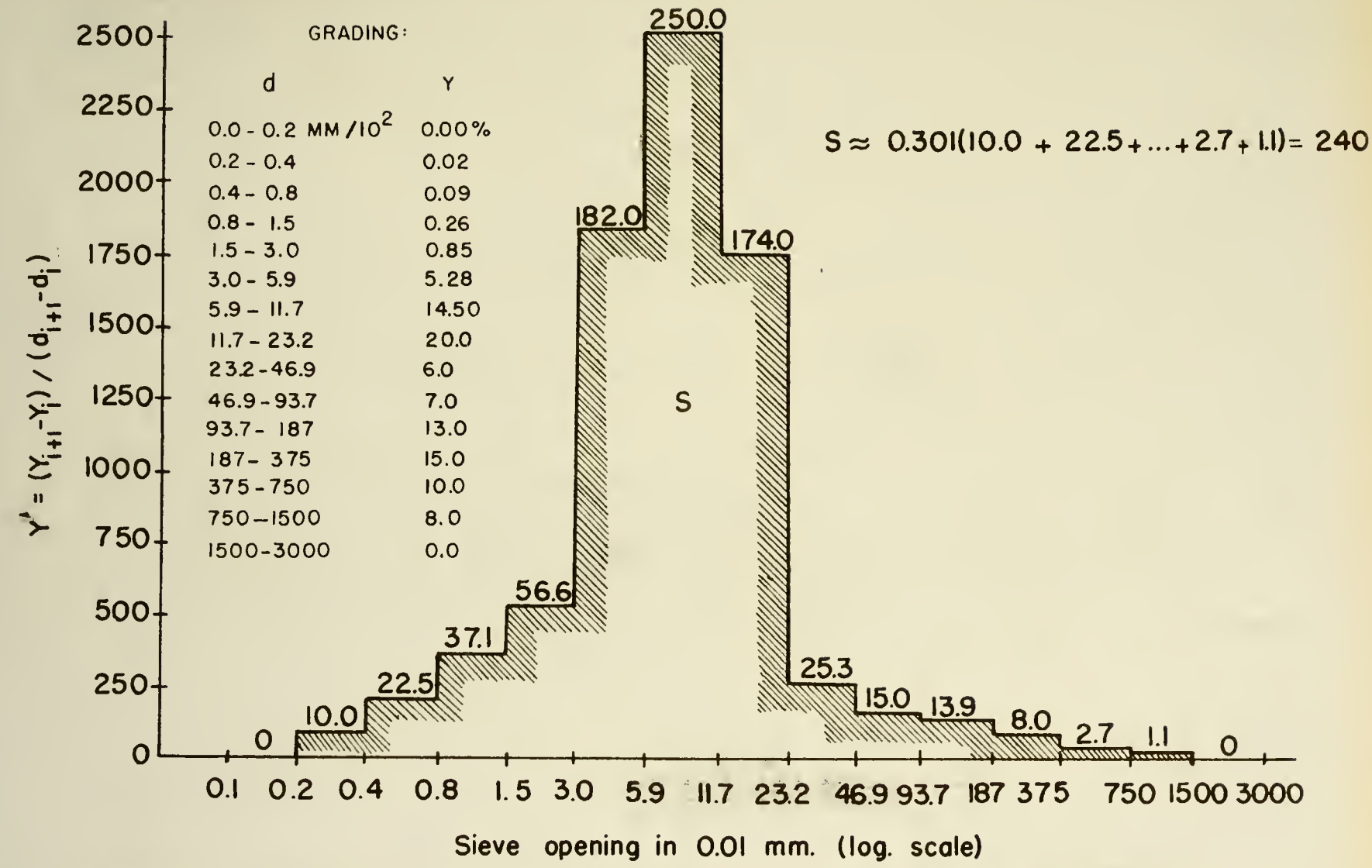

Figure 4. Second Grading Area as a Characteristic of the Specific Surface

paste forming a film around particles which are the same order of magnitude as the particles of cement themselves. Much of the material passing the No. 100 sieve is not much larger than the larger particles of the cement. These small particles act as void fillers but can hardly be held to require a surface film for their lubrication in the mass."

This is a logical point of view and it can be adopted. Other investigators have also found that a small change in the quantity of the smallest particles affects neither the workability nor the strength of concrete (25). However, this problem is one of the cardinal points of the specific surface area theory because a slight change, say half a percent, in the quantity of the particles passing the No. 200 sieve can cause a change of 20-50 percent in the "true" specific surface area of the aggregate. Therefore, several tests relating to this question were performed by the writer. The test results are presented in Table 2. It is seen that these results are in accordance with the statements above.

Sphere as Calculated Particle Shape

The assumption that the shape of particles is spherical or cubic is merely a simplification. Its use is excused by the fact that the calculation of surface area is simplest when done in this way. The specific surface area calculated on the basis of a sphere is always less, often considerably less, than the true surface area of the aggregate. The users of surface area theory were challenged many times because of

this fact. This is an important objection to the Edwards - Young theory since this method involves computation of the quantity of cement paste that is necessary to surround the particles. It is obvious that in this case the actual value of the surface area is the decisive factor.

However, if the numerical value of the specific surface area is used to characterize the variance of the particle-size distribution instead of to calculate the quantity of cement paste, then the assumption of spherical shape does not carry much weight because the value of s is considered not as an actual surface area but as a characteristic of particle size distribution. Thus, it is unquestionable that some error is made.[7] A mitigating circumstance is that the spherical shape is also assumed to apply to the fundamental characteristic of grading, namely, the particle-size since the acceptance of the measurement of the particle-size by a sieve test implies such a shape. Nevertheless, it would be desirable to have higher accuracy in the calculation of the specific surface area than is possible with presently-available methods.

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[7]This is increasingly true if the particle shape cannot be considered the same in the entire aggregate. Such a case occurs when sand is used as the fine aggregate and crushed stone as the coarse aggregate.

TABLE 2

COMPARISON OF CONCRETES MADE WITH SAND IN A NATURAL STATE AND WITH WASHED SAND

Aggregate Grading: Continuous; D = 1"; m = 5.3; s = 3.0 m^2/kg

Mix proportion: 1:6 by weight

Water-cement ratio: $\frac{W}{c}$ = 0.57 by weight

Test[6]	Air Entrained				Non Air Entrained			
	Natural Sand		Washed Sand		Natural Sand		Washed Sand	
Slump (in.)	2.0 2.0	2.0	1.75 2.0	1.88	3.0 2.75	2.88	2.5 2.75	2.62
Bleeding (cm^3)								
20 min.	13.5		7		12		10	
100 min.	67		71		59		68	
Strength (psi) 7-day	4780 4670 5170	4873	4750 4800 4050	4530	4260 3780 3730	3910	4110 4510 4560	4390
Segregation (%) top	-		-		43.8; 38.5; 43.3		43.5; 46.8; 46.0	
bottom	-		-		51.5; 48.7; 46.1		49.7; 54.0; 52.4	
Segreg. Fact.	-		-		1.17; 1.26; 1.07		1.14; 1.15 1.14	
Cement factor (bgs/cu.yd.)	5.82		5.73		5.70		5.69	
Unit Weight (lbs/cu.ft.)	153.1		150.9		150.3		149.9	

[6]The testings methods will be presented later (see pp. 66).

Comparison of the Fineness Modulus and Specific Surface Area

Sands and gravels of natural origin seldom have extreme gradings, for instance, gap gradings; therefore in these cases the decrease of the fineness modulus often corresponds to an increase in the specific surface area. Consequently, there is usually some empirical relationship between the values of m and s for a given pit.

By reason of this apparent relationship between values of m and s, some researchers believed at first that the fineness modulus and the specific surface area can be expressed in terms of each other, at least in an approximate way (21) (26). However, Abrams emphasized that, in general, this is not true and that the fineness modulus and specific surface area are entirely independent of one another (27). Since then several researchers have pointed out this fact (28), and the demonstrations of this paper also show this.

Nevertheless these two factors do show some interesting analogies:

a) Each of these factors corresponds to a statistical parameter: the fineness modulus to the average, the specific surface area to the variance of a particle size distribution. Casually, it is not an accident that both of these numerical characteristics of grading are proportional to the two most ordinary statistical parameters. That would be rather surprising if the empirical character-

istics of the particle size distribution of a grading were not connected with the theoretically proved, statistical characteristics of distribution.

b) Both the fineness modulus and the specific surface area of an aggregate divided into fractions can be calculated by similar formulae:

$$m = \sum_{i=1}^{n} \frac{p_i m_i}{100} \qquad s = \sum_{i=1}^{n} \frac{p_i s_i}{100}$$

c) Both the fineness modulus and the specific surface area can be expressed by the same type of Stieltjes-integrals:

$$m = 0.0332 \left[100 \log (10D) - \int_{0.1}^{D} f(d)\, d\log d \right] =$$

$$= 0.0332 \left[100 \log (10D) - 0.4343 \int_{0.1}^{D} \frac{f(d)}{d}\, dd \right]$$

$$s = \frac{6}{100\gamma} \int_{d_{min}}^{D} f'(d)\, d\ln d =$$

$$= \frac{6}{100\gamma} \int_{d_{min}}^{D} \frac{f'(d)}{d}\, dd$$

d) Both the fineness modulus and the specific surface area are related to an area in a similar system of co-ordinates. From these areas F and S, m and s

can be simply calculated (Figure 5):

$$m = \frac{F}{30.1} \qquad s = \frac{13.82}{100\gamma} S$$

Thus, both of these characteristics of grading can be determined by a summation of ordinates of grading curves, which can be drawn easily on the basis of a sieving test.

The basic difference between the fineness modulus and the specific surface area is that the fineness modulus is mainly influenced by larger particle sizes; on the other hand, the specific surface area gives expression mostly to the effect of small particles. In other words, this means that if the emphasis is on the small particles (e.g., in the case of the evaluation of cement fineness), then the specific surface area, as a single number, is more suitable to evaluate the grading than is the value of the fineness modulus; however, if the large particle sizes are important (e.g., grading of gravel), the use of the fineness modulus seems to be better.

But now we arrive at the question that has been frequently asked for 40 years: which factor is more suitable to evaluate a grading with respect to concrete technology, the fineness modulus or the specific surface area? The author believes that the fineness modulus is a more reliable characterization of the aggregate grading than is the specific surface area. However, such a judgment is not

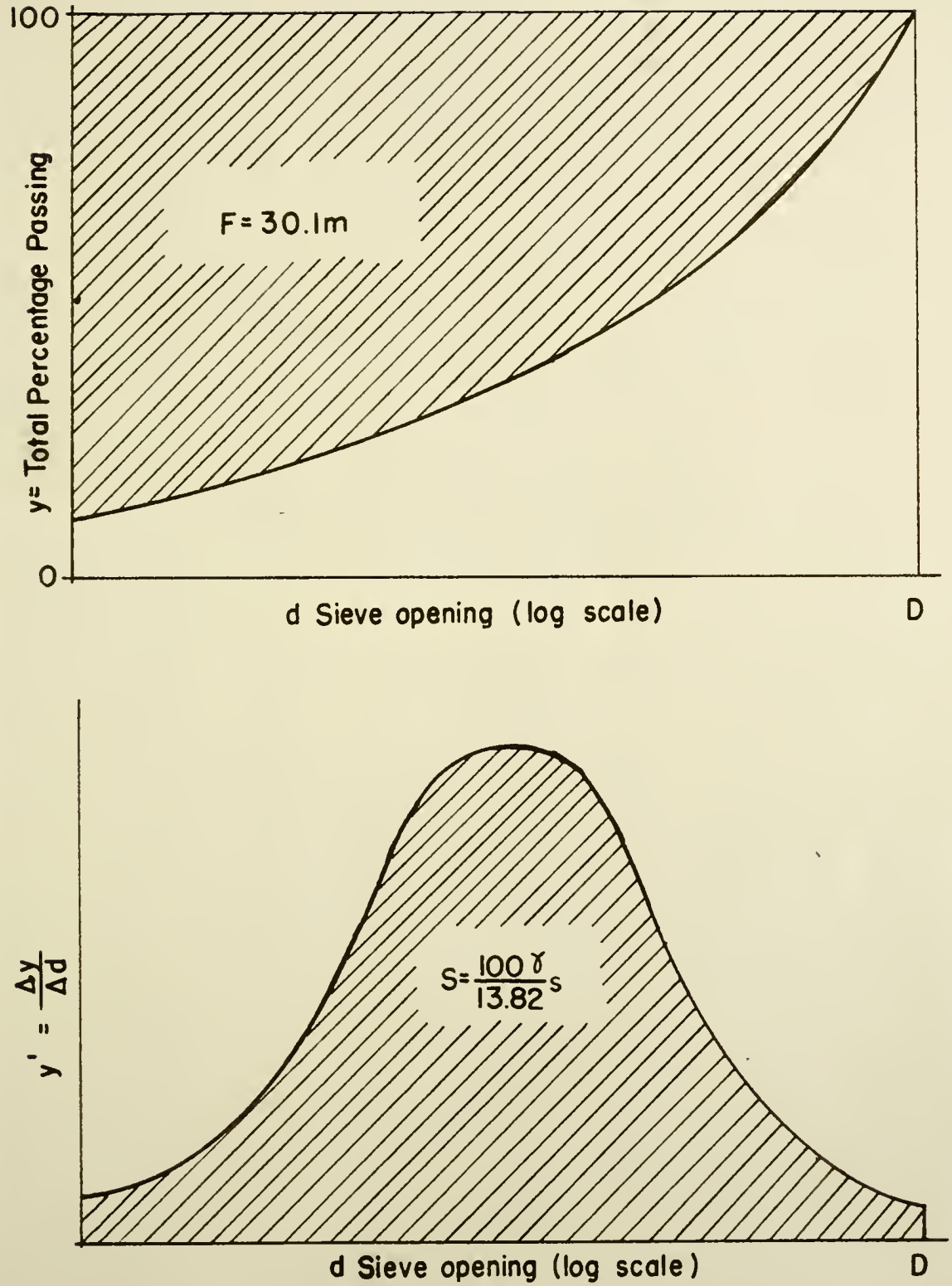


Figure 5. The Fineness Modulus and Specific Surface as characterized by Areas

really necessary because there is a natural solution to this problem. Since the fineness modulus is an average value concept, and the specific surface area is a variance concept, both factors should be used simultaneously.

More exactly, the author's proposal is the following: For an evaluation of aggregate grading, the maximum particle size, the fineness modulus, and the specific surface area should be used simultaneously, irrespective of the particle-size distribution.

This proposal, its justification, and support by experimental results are the subject of the rest of this paper.

SIMULTANEOUS USE OF THE FINENESS MODULUS AND SPECIFIC SURFACE AREA

As it has been discussed previously, the fineness modulus is not always an entirely dependable tool with which to evaluate an aggregate grading. On the basis of fineness modulus, sometimes gradings can be qualified as good when they are really useless for concrete purposes. The fineness modulus is a necessary, but not a sufficient condition.

In other words, if the grading is good, the value of the fineness modulus is always near the optimum. If, however, the fineness modulus of a grading equals the optimum value, this fact does not necessarily indicate that the grading is excellent or even fair because the adequate workability of a grading is not assured by just the numerical value of the fineness modulus. Therefore, one group of researchers refused the fineness modulus concept and, instead they suggested other characteristics such as maximum density, specific surface area, avoidance of particle interference, etc.

Other researchers, appreciating the good properties of fineness modulus, proposed to use other characteristic values in addition to the numerical value of the fineness modulus. Some of these additional proposals are briefly discussed in the following paragraphs.

Attempts to Improve the Fineness Modulus Method

Examination of Portions of the Grading with Fineness Modulus

Stanton and Bartel write (19) about the maximum values of the fineness modulus which "can be attained only with well graded aggregates. All portions of the combined aggregate from 'zero' size to the various intermediate sizes should have fineness moduli not greater than permitted for those intermediate sizes, if workability is to be assured." In another place they say (19): "It is apparent that a mix which is harsh for 3/4 inch aggregate will not be rendered less harsh by adding larger size material. It should be determined that the maximum allowable fineness modulus is not exceeded by that portion of the aggregate finer than any size, if workable concrete is to be insured."

This is a logical conclusion. Nevertheless, one can raise two objections to it. First the condition solves only one side of the problem because in this manner just the very coarse gradings are eliminated. The extremely sandy gradings are not considered. Second, the practical utilization of the Stanton-Bartel condition is complicated. If the grading is controlled at different particle sizes, the cement content (which should be taken into account in each case separately for the maximum permissible value of fineness modulus) changes point by point. This is obvious because, according to the above method, we should consider as removed from the original amount of aggregate the particles which are bigger than the

controlled particle size. The change of cement content should be calculated at all controlled points. This process is too complicated.

Limits for Fine Particles

In Palotas' opinion (29), a necessary condition for good workability, in addition to the adequate numerical value of the fineness modulus, is that the grading should consist of particles of various diameters; the permissible upper limit of the minimum size of aggregate is 0.3 mm, that is No. 50 sieve.

These are simple and practical rules. However, one can raise two objections again. First, these rules eliminate only the too coarse gradings again. The extremely sandy gradings are not even touched. Second, an essential feature of the fineness modulus is that it characterizes gradings in an over-all, rather than a detailed manner. Palotas' conditions refer, however, to the details of the grading.

Unfortunately, it does not bring any great improvement if, in addition to the above conditions, a lower limit and an upper one are specified for the amount of particles, say, of 0-1 mm (passing No. 16 sieve). If these limits are too large, unsuitable gradings can occur; and if they are too narrow, many usable gradings are excluded. Besides, such conditions restrain the shape of the grading curve still more.

Fulton's Proposal

On the basis of the grading diagram, Fulton says (30) that the fineness modulus "equals the average width of the diagram" in the system of coordinates of Figure 1. Then he says: "The distribution of the particle sizes, in so far as they deviate from the average size or fineness modulus, is indicated by the standard deviation."

"Where p_1, p_2, p_3, etc. are the percentages that occur within certain groups or size fractions, we may write

$$S.D = \sqrt{\frac{p_1(x_1-\bar{x})^2 + p_2(x_2-\bar{x})^2 + \ldots}{100}}$$

in which x_1, x_2, etc. are now the mean values of each group." S.D. means the standard deviation and $\bar{x}$ is the fineness modulus of grading.

Fulton does not prove his statements and even his wording is not precise. For example, the fineness modulus is not "equal" to the "average width of the diagram," but it is proportional to the logarithmical average of particle sizes, etc. Quite independent of this fact, it is unfortunate that Fulton has not published any test results regarding his theory because it is not possible to form an opinion of its accuracy and practicability.

The Specific Surface Area as an Addition Value

Essentially in the same manner as in Fulton's proposal, the author also devised a method to improve the fineness modulus method (10).

Accordingly, besides the fineness modulus (m), the numerical value of the specific surface area (s) is used as an additional value to evaluate a grading.

The writer believes that this method is as suitable for gap as for continuous gradings and that its precision is not less than that of the method using sieve limit curves for continuous gradings. This method is supposed to unite the advantages of the fineness modulus method and those of the method of sieve curves.

The practical implications of such a method of characterizing gradings are that materials heretofore considered unsuitable for making concrete could be used advantageously. An example of such use is the case of areas in Indiana that have only dune sands as the local aggregate. This sand is a uniformly fine material, mostly between the 30 and 100 mesh sieves. A typical grading curve for this material is given as D-1 in Figure 6 (31). Also shown are three pairs of curves, each pair having a different maximum size D. One curve of each pair is a typical continuous grading while the other is a grading using only the 30-100 mesh dune sand as the fine aggregate. These gap gradings have been constructed to have the same D, m, and s as the corresponding continuous gradings. As previously mentioned, there is reason to think that the gradings in each pair are equivalent in terms of the properties of the plastic and hardened concrete made with them.

The rest of the Third Part is a detailed discussion of this theory.

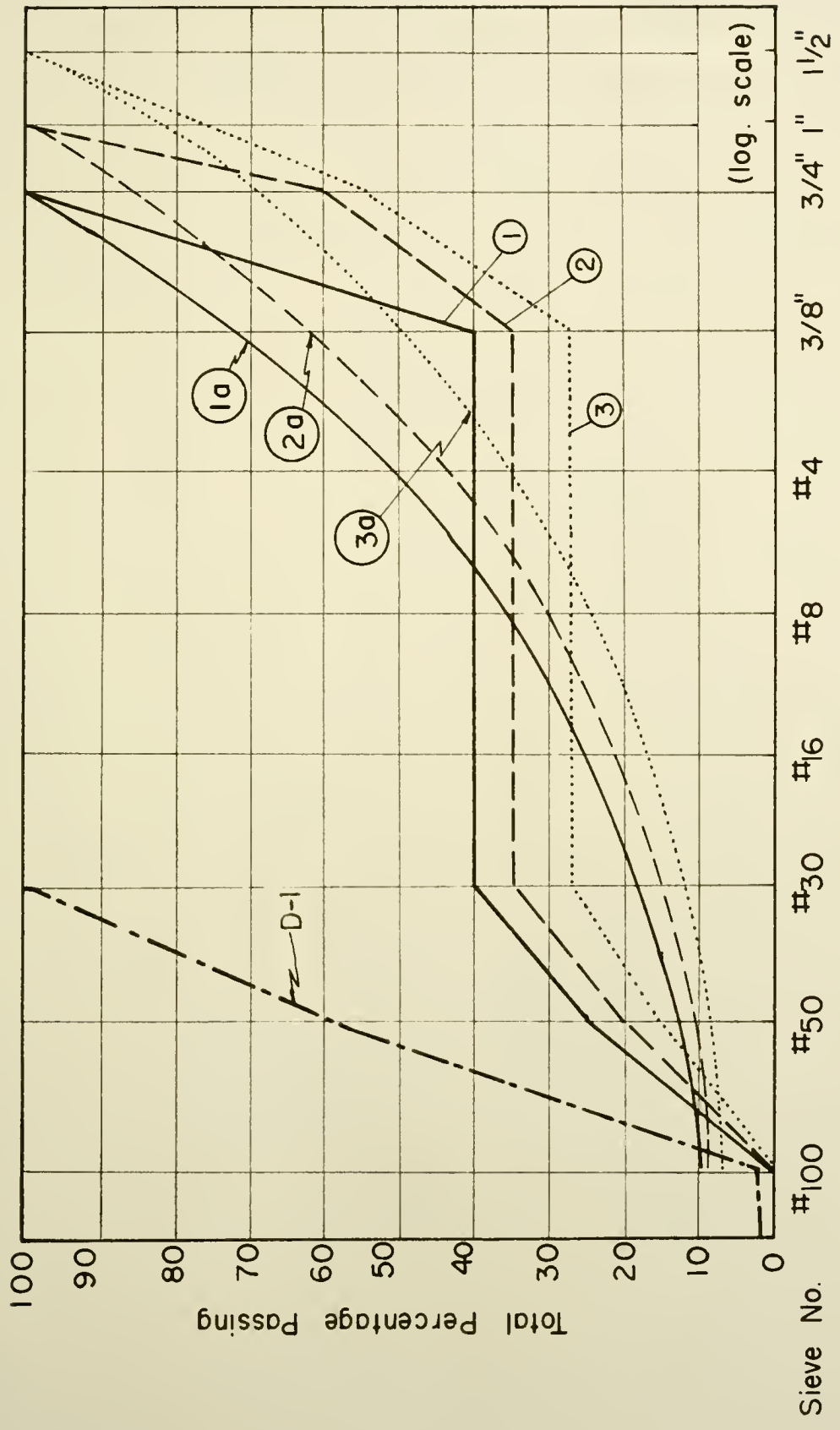


Figure 6. Three Gap Gradings Utilizing a Typical Dune Sand

Theoretical Analysis of the Proposal to Use the Fineness Modulus and Specific Surface Area Simultaneously

It is the author's opinion that three factors are sufficient for an evaluation of grading, the maximum particle size (D), the fineness modulus (m), and the specific surface area (s), irrespective of the particle size distribution.

This proposition includes two statements:

1) Gradings of identical fineness modulus can be distinguished by means of specific surface area. Among the gradings of identical values of m, there is one value of s_o that is optimum, i.e., that results in, say, the maximum strength. Either lower or higher values than the optimum one result in lower strengths, and, as will be presented, the greater the deviations from the optimum, the lower the strength.

2) Gradings of identical fineness modulus and specific surface area with the same mximum size are equivalent, irrespective of the particle size distribution; thus for a given mix design and materials, gradings that have the same values of D, m, and s should produce concrete with the same properties in both the plastic and hardened condition, regardless of the details of the gradings.

Statistical Reasons

It has been proved previously that the fineness modulus is an average-concept and the specific surface area is a variance concept.

The fact that every particle size distribution of the aggregates of identical fineness modulus is not equally satisfactory, even in the case of identical maximum particle sizes, can be explained statistically. Namely, no consideration of variance is permitted even in the instance of particle size distributions of identical average and upper limits. If the variance is too small, the aggregate tends to be a type of one size, i.e., its void content is high; therefore, its workability is not sufficient and it is practically impossible to make dense, strong concrete. On the other hand, if the variance is too large, the aggregate requires more mixing water to form workable concrete, and therefore the concrete is of poorer quality. Essentially, one gets a similar result if the application of the specific surface area concept is tested by itself.

Thus, it seems expedient to take into account the variance of the particle size distribution in order to distinguish among various gradings more accurately. In the following the fineness modulus is used to characterize the average size and the specific surface area is used to characterize the variance.

If the grading is continuous and the minimum particle size is 0.1 mm or less, an adequate, not too small--not too

high value of the variance, in general, results as a consequence of the continuity of the grading. Therefore, the fineness modulus is usually satisfactory alone to judge such gradings, but only such gradings.

Concrete Technology Analysis

Another explanation of the fact that one cannot expect a reliable evaluation of every grading by means of either fineness modulus or the specific surface alone is given below.

Let an aggregate be considered to consist of n different fractions. Then the following relationship is valid for the m_a fineness modulus of the aggregate:

$$m_a = \sum_{i=1}^{n} p_i m_i = p_1 m_1 + \cdots + p_n m_n$$

Some of the m_i values are shown in the m column of Table 3.

This equation shows how changes in the fractions influence the m_a fineness modulus of the aggregate. Thus, the change in the quantity of particles passing the No. 100 sieve does not influence the value of m_a directly since the fineness modulus of this fraction is equal to zero by definition. The change of $\Delta p = \pm 0.1$ (10%) by weight in the quantity of the fraction passing the No. 50 sieve but remaining on the No. 100 sieve causes only a negligible difference of $\Delta m'_a = \pm 0.1$ in the value of m_a; on the other hand

the same change of $\Delta p = \pm 0.1$ in the fraction $1\frac{1}{2}$" - 3" causes a significant difference of $\Delta m_a" = \pm 0.9$. Accordingly, the coarse particles have a decisive influence on the fineness modulus. However, the test results show that the changes in the quantity of fine particles have a greater effect upon the properties of concrete than the changes in the coarse particles (32).

Consequently, the fineness modulus expresses the effect of coarse aggregates more strongly than is warranted by experience. This effect occurs to the detriment of fine aggregates.

Similarly, it is not expected to characterize the aggregate grading in a satisfactory manner from the value of specific surface alone. But here the situation is just reversed. The specific surface area characterizes the fine particles of an aggregate very well. It cannot show, however, the effect of coarse particles because the s_i values of the various coarse aggregate fractions are too small. (See column s in Table 3.)

This defect can be eliminated by the simultaneous application of fineness modulus and specific surface area. Both the fine and the coarse particles of an aggregate can thus be taken into account. The fineness modulus controls the coarse part of the aggregate, the specific surface area controls the fine part. So, if the fineness modulus of an aggregate is near the optimum one and, in addition, if the

specific surface area of the grading is adequate, then the amount of mixing water necessary for concrete made from the aggregate in question will not be excessive, and, in addition, sufficient workability and density are assured.

Accordingly, the writer's proposal seems to be theoretically reasonable. Below, it is shown how test results support this theory.

Tests with Mortar

Palotas' Tests

Several test series performed by Palotas (29) can be used to control the foregoing theory. Figure 7 illustrates one group of the data. The evaluation of these test results was presented in a previous paper (10).

This chart shows the 28-day compressive strength, the flow and the unit weight values of cement-mortars made with river sand, as a function of the specific surface area of the mineral aggregate. Various sand gradings were composed so that their fineness modulus was the same. Maximum particle size was D = 5 mm (≅1/5 in.). The cement-content was 9 bags/cu yd; the water-cement ratio was 0.53 by weight. The flow of the fresh mortars was determined by the small flow table specified in the German standards (DIN 1164). Thus the presented flow data mean the diameters of the spread mortar specimens after the flow table was raised and dropped twenty times. The complete description of these tests (sieve curves, etc.) can be found in the original paper (29).

TABLE 3

VALUES OF THE FINENESS MODULUS AND SPECIFIC SURFACE AREA FOR VARIOUS AGGREGATE FRACTIONS

Particle Size		Fineness Modulus m_i	Specific Surf. Area s_i m^2/kg
Sieve No.	Sieve Opening mm		
No. 200-No. 100	0.07 -0.149	0	21.33
No. 100-No. 50	0.149-0.297	1	10.67
No. 50-No. 30	0.297-0.59	2	5.33
No. 30-No. 16	0.59 -1.19	3	2.66
No. 16-No. 8	1.19 -2.38	4	1.33
No. 8-No. 4	2.38 -4.76	5	0.67
No. 4- 3/8"	4.76 -9.52	6	0.33
3/8" - 3/4"	9.52-19.05	7	0.17
3/4" - 1½"	19.05-38.1	8	0.08
1½" - 3"	38.1 -76.2	9	0.04

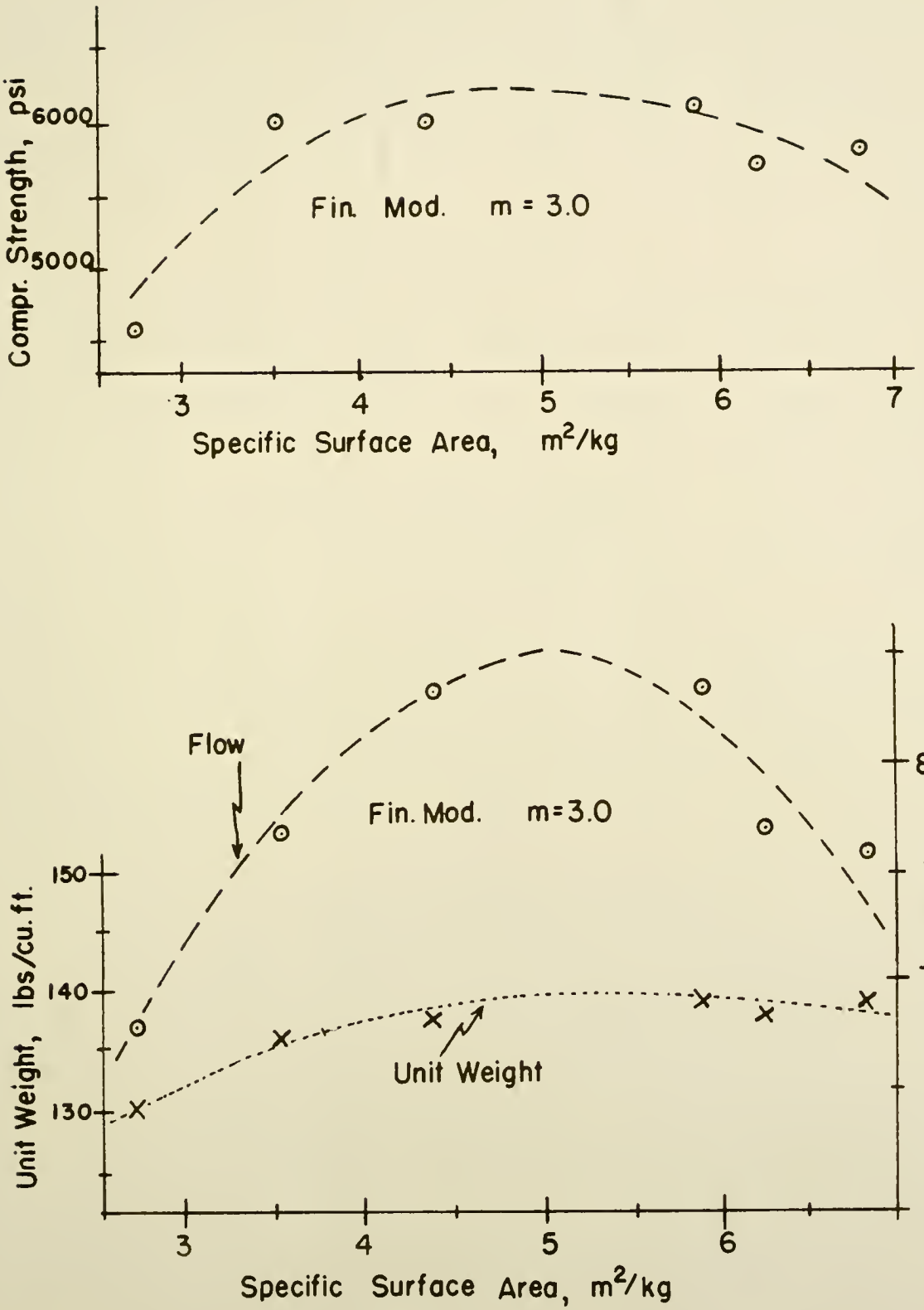


Figure 7. Strength, Flow, and Unit Weight of Concrete as a Function of the Specific Surface Area of the Aggregate. (Data from Palotas (29).)

These test results show that gradings having the same maximum particle size and fineness modulus can be ranked on the basis of specific surface area in respect to compressive strength, unit weight, and flow.

Writer's Tests (11)

For these tests three groups of river sand were used. Each group had the same fineness modulus, but within the groups the specific surface area of the gradings varied. The specific surface area of the fraction 0-0.2 mm was taken into account as $s \cong 20\ m^2/kg$. The flow of all mixtures was about 4" according to DIN 1164. As an example, one group of gradings is shown in Figure 8.

Figures 9 and 10 show the 7-day strength for this group as a function of specific surface area. The complete description of these tests can be found in the original paper (11).

These results also support the theory.

Tests of Hardened Concrete

In these tests, performed by the writer (11), river sand and gravel were used. The specimens were prepared according to the Hungarian standard (MNOSz 934).

Several grading groups were tested. Within each group the D, m, and s parameters were the same, but the particle size distributions differed in details. Besides a continuous grading, every group contained several different gap

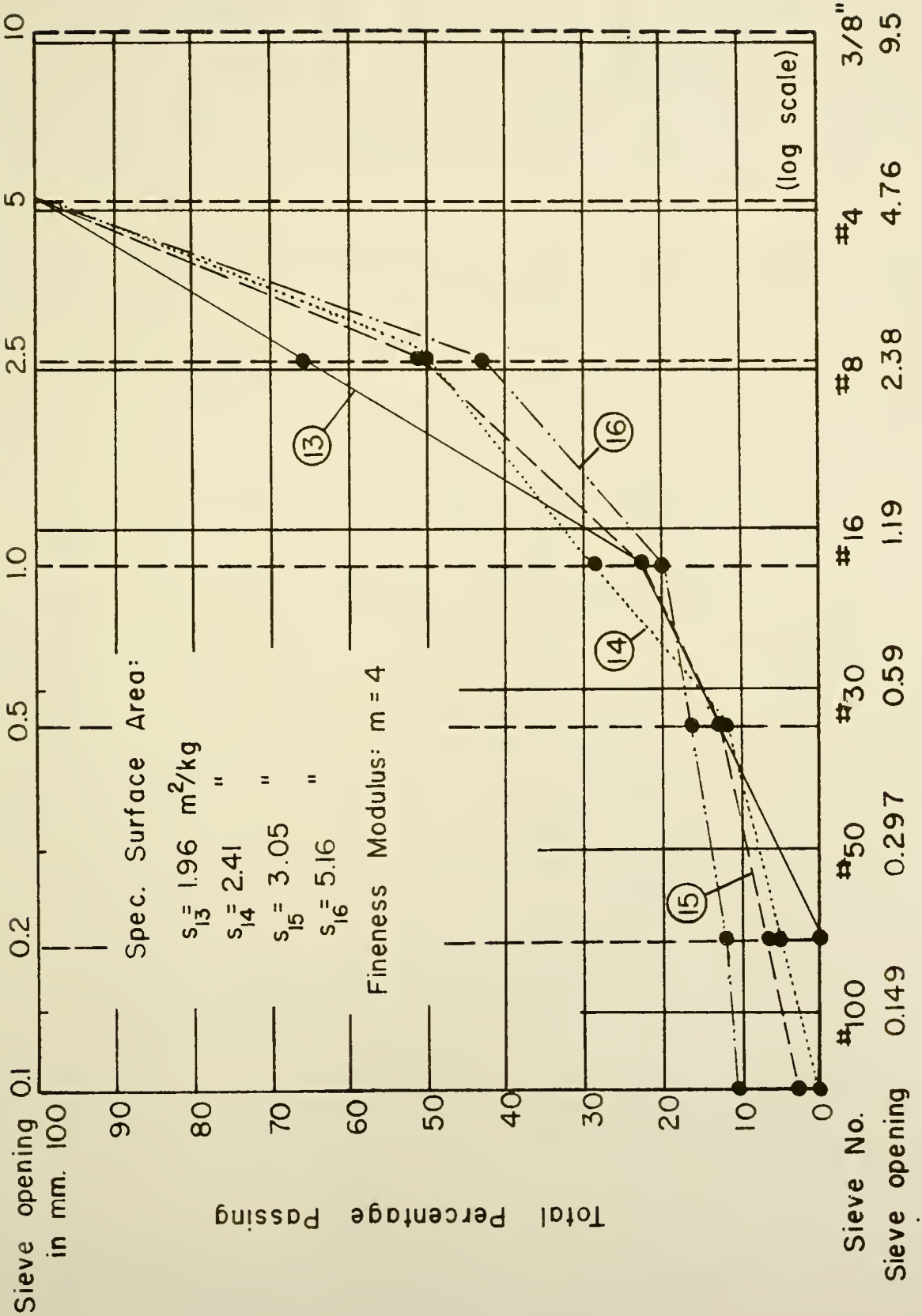
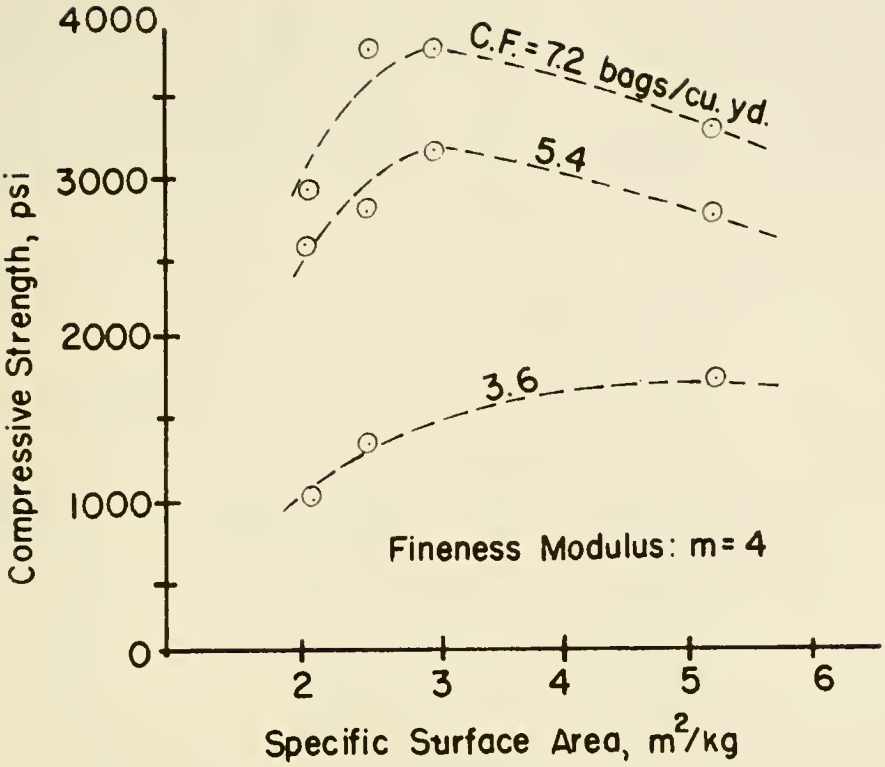
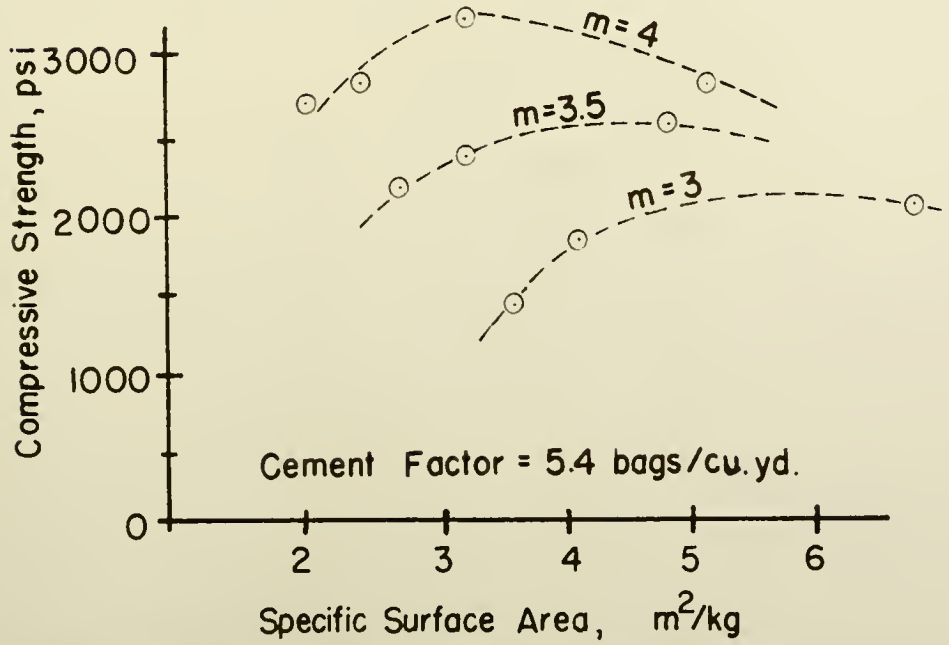


Figure 8. Gradings Having the Same Values of D, and m but Different Values of s





Figures 9-10. Compressive Strength of Concrete as a Function of Cement Factor, Fineness Modulus, and Specific Surface

gradings. Figure 11 shows an example for the group of $D = 1\frac{1}{2}"$, $m = 5.8$, and $s = 2.1$. The test results of concretes made with these gradings are presented in Table 5.

TABLE 5

7-DAY TEST RESULTS OF CONCRETES MADE WITH GRADINGS 7, 8, AND 9 (11)

No. of Grading	Unit Weight lbs/cu. ft.	Compressive Strength, psi
7	148	2890
8	146	2880
9	146	2790

The results of these experiments show that the gradings having the same D, m, and s parameters, are equivalent in respect to compressive strength and unit weight, irrespective of the details of particle-size distribution.

Tests of Freshly Mixed Concrete

In the results presented above, the properties of hardened concrete were emphasized. It is necessary to check the suggested theory of grading evaluation for the properties of fresh concrete. The investigation of those properties was desired which are considered more disadvantageous for gap gradings than for continuous gradings. These properties are consistency, bleeding, and segregation.

The purpose of these tests was, again, to support the hypothesis that gradings having the same D, m and s values

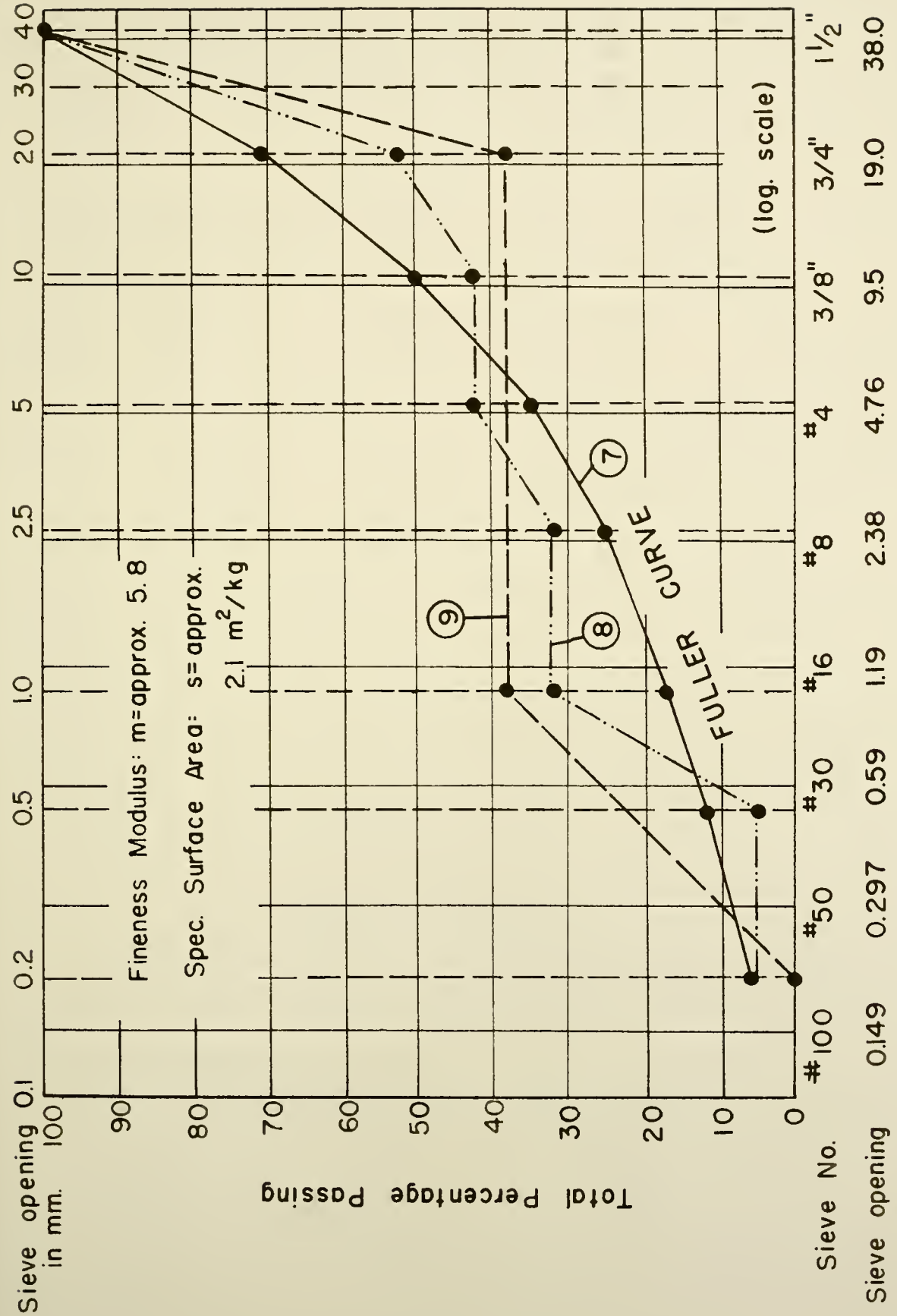


Figure 11. Gradings Having the Same D, m, and s Values I

exhibit practically the same concrete making properties irrespective of the details of the grading.

Materials

The same Type I portland cement was used in the entire test series. The fine part of the mineral aggregate was sand from Lafayette, Indiana. The coarse aggregate was crushed limestone from Greencastle, Indiana.

Two different maximum particle sizes were used. Three different gradings were planned for each maximum size: namely, one of them was a continuous grading (marked by C), another was of one gap (marked by O), and one had two gaps (marked by T). The values of fineness moduli and specific surface areas were identical for the C, O, and T gradings within a group. These gradings are shown in Figures 12 - 14. The particle size distributions of the gap gradings, mainly those of the O types, were purposely made extreme to check the validity of the theorem.

Experimental Design

Concrete mixtures were made with two different cement contents and two different consistencies both with and without air entrainment.

The factors and levels were the following:

Factors		Levels	
Maximum particle size:	d	d_1	3/4"
		d_2	1"

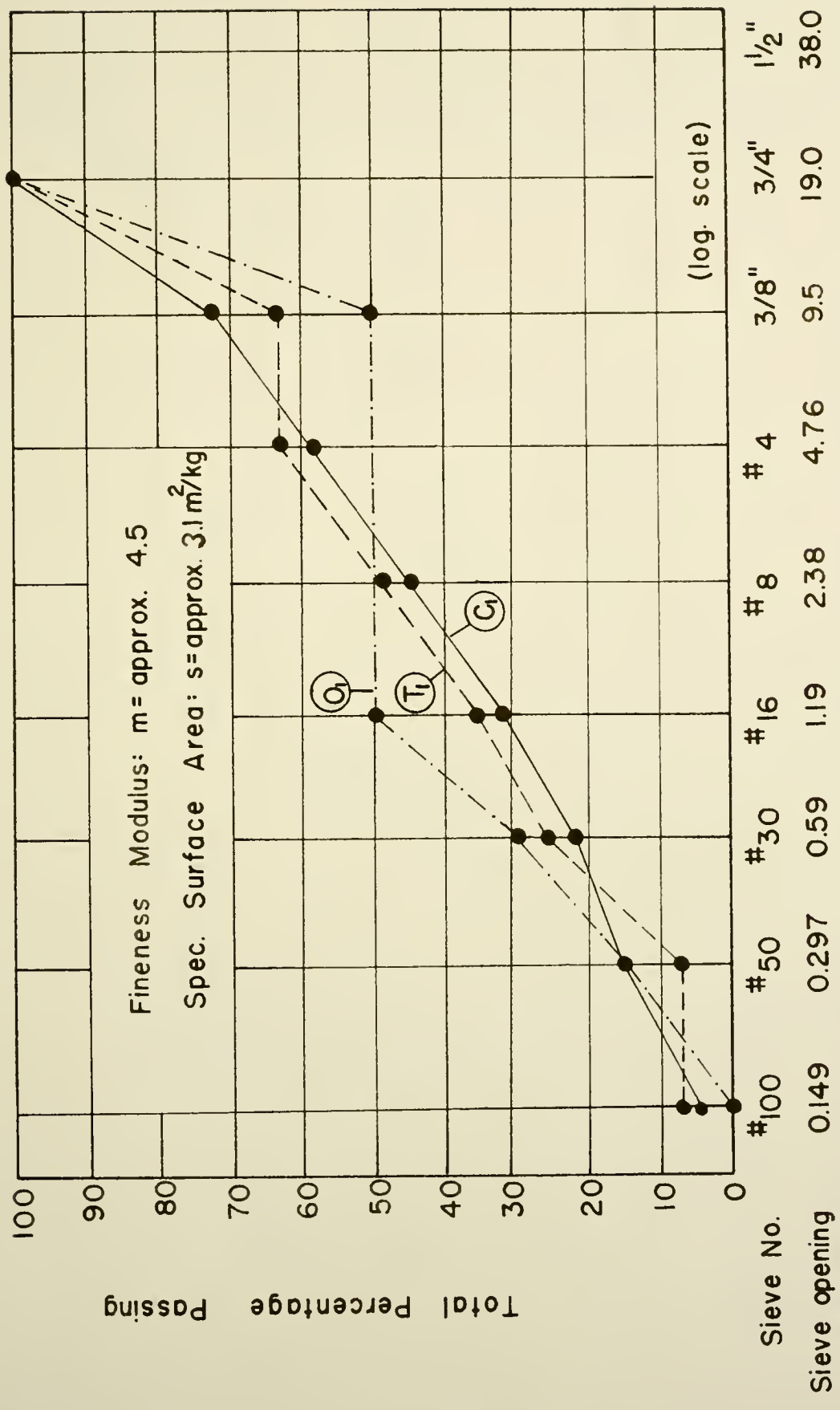


Figure 12. Gradings Having the Same D, m, and s Values II

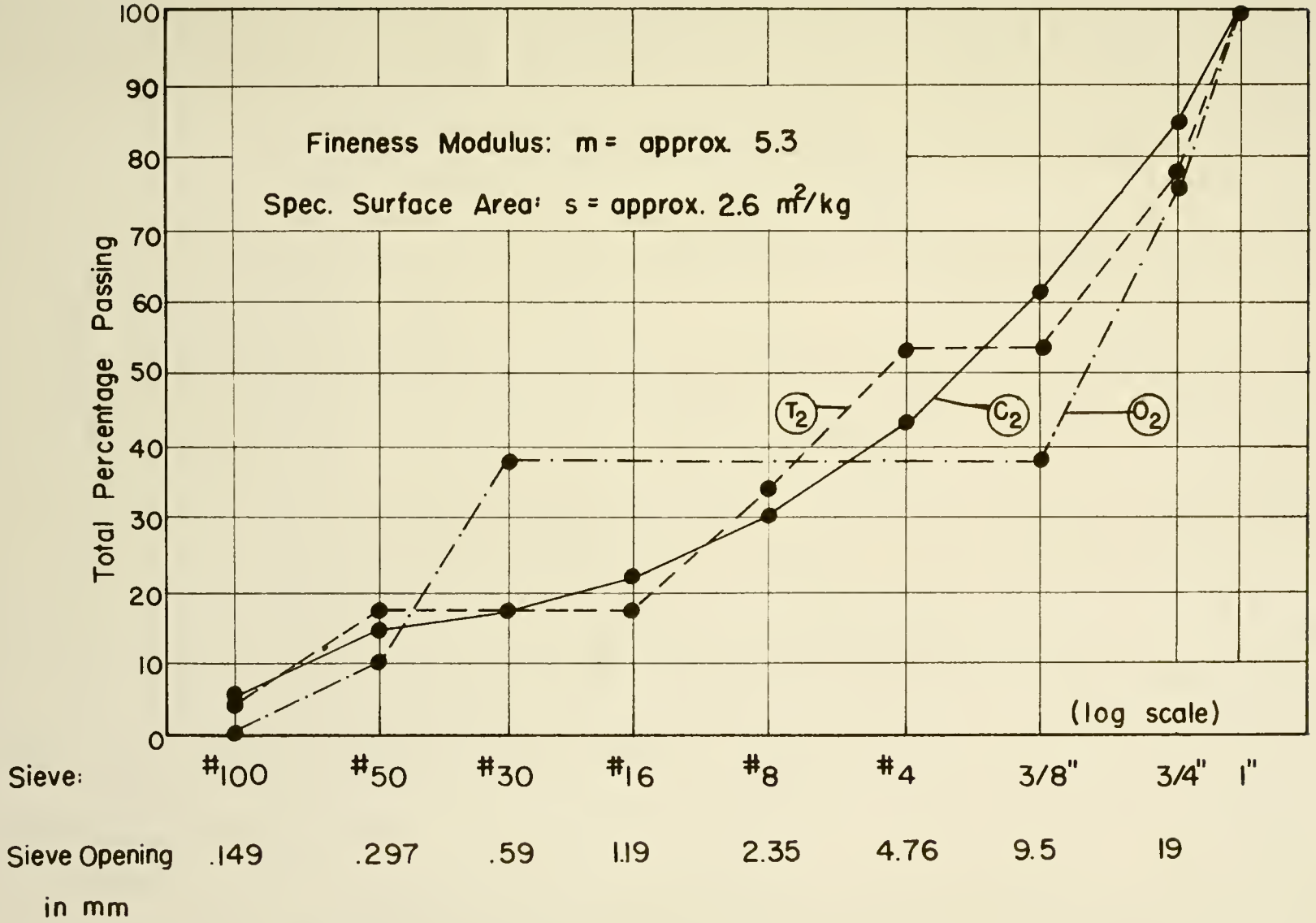


Figure 13. Gradings Having the Same D, m, and s Values III

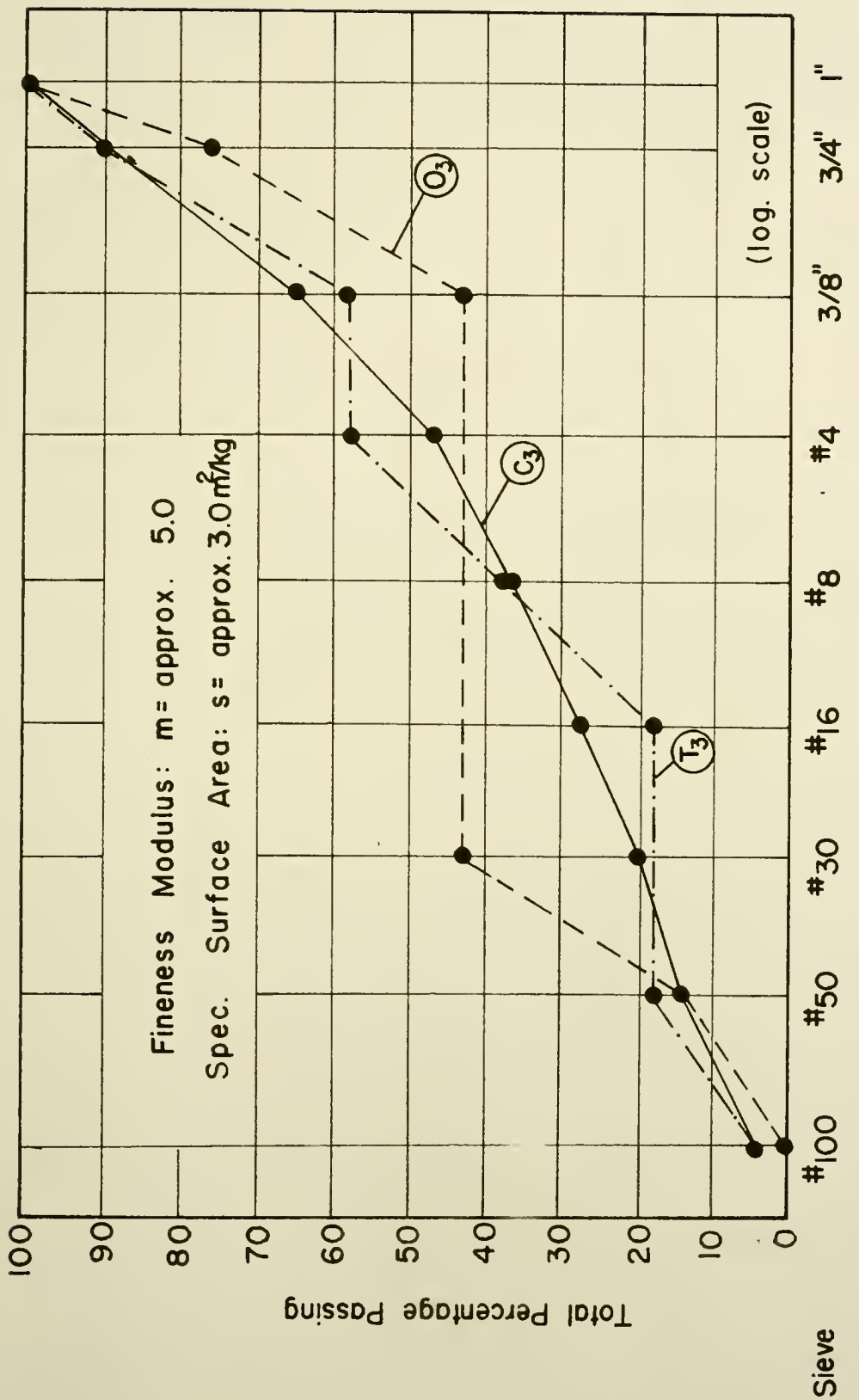


Figure 14. Gradings Having the Same D, m, and s Values IV

Factors	Levels	
Air content : a	a_1	without air entrainment
	a_2	with air entrainment
Cement content : c	c_1	4.5 bag/cu yd
	c_2	6.0 bag/cu yd
Consistency : w	w_1	water for stiff consistency
	w_2	water for loose consistency
Grading : g	g_1	continuous, C
	g_2	one gap, O
	g_3	two gap, T

The experimental layout is shown in Table 6.

The type of experiment is a fractional factorial one with a half replication. The number of treatment combinations (mixtures) is

$$\frac{2^4 \times 3}{2} = 24$$

The selection of the treatment combinations was based on two considerations: (1) Every tested concrete composition without air entrainment is repeated also with air entrainment, and (2) the g factor is not confounded.

In the presented experimental layout only the CDW term is confounded with the block.

The purpose of this experiment was to test the hypothesis that the C, O, and T gradings are equivalent with respect to concrete making properties. This is called the equivalency hypothesis. The test of this hypothesis is performed by

TABLE 6

EXPERIMENTAL LAYOUT

Maximum Particle Size	Air Content	Cement Content	Consistency	Grading
		c_1	w_1	C O T
	a_1			
		c_2	w_2	C O T
d_1				
		c_1	w_1	C O T
	a_2			
		c_2	w_2	C O T
		c_1	w_2	C O T
	a_1			
		c_2	w_1	C O T
d_2				
		c_1	w_2	C O T
	a_2			
		c_2	w_1	C O T

analysis of variance and, in addition, by individual comparisons.

The following tests were performed on each mixture:

Slump	ASTM Designation: C 143-58
Flow	ASTM Designation: C 124-39
Bleeding	ASTM Designation: C 232-58
Segregation	No standardized test
Unit Weight	ASTM Designation: C 138-44
Compressive Strength	ASTM Designations: C 192-57 and C 39-56T

The statistical computations are presented in the next part.

Tests and Test Results

Slump Test. This test was performed according to the ASTM Specifications. Two-two measures were made from each mixture. The results are presented in Tables 7 - 8.

Flow Test. This test was performed according to the ASTM Specifications. Two-two measures were made from each mixture. The results are presented also in Tables 7 - 8.

Bleeding. This test was performed according to the ASTM Specifications. One-one measure was made from each mixture. The testing results are presented in Tables 9 - 10.

Test of Segregation. There is no standardized method for this test. The following method was used. A metal container of about 1/4 cu. ft. was filled with fresh con-

TABLE 7

CONSISTENCIES OF CONCRETES MADE WITH GRADINGS C, O, AND T AND WITH D = 3/4 IN.

Run No.	Grading	Air Entrained?	w/c	C bags/cu yd	Slump, in.		Flow, %	
13	C	No	0.80	4.5	1.25 1.5	1.38	94.3 85.0	89.6
15	O	No	0.80	4.5	1.25 1.25	1.25	88.8 87.9	88.3
16	T	No	0.80	4.5	2.75 2.75	2.75	117.5 116.2	116.9
14	C	Yes	0.80	4.5	4.25 3.0	3.62	122.5 112.5	117.5
18	O	Yes	0.80	4.5	2.0 1.5	1.75	94.1 103.8	98.9
17	T	Yes	0.80	4.5	4.25 4.5	4.38	131.7 120.9	126.8
19	C	No	0.67	5.8	6.25 5.25	5.75	141.2 128.8	135.0
21	O	No	0.67	5.8	3.5 3.5	3.50	126.3 128.8	127.6
20	T	No	0.67	5.8	6.5 6.75	6.62	146.0 140.0	143.0
23	C	Yes	0.67	5.8	7.0 6.75	6.88	152.9 147.1	150.0
22	O	Yes	0.67	5.8	6.5 6.0	6.25	144.2 155.0	149.6
24	T	Yes	0.67	5.8	7.25 7.5	7.38	142.1 150.4	146.2

TABLE 8

CONSISTENCIES OF CONCRETES MADE WITH GRADINGS C, O, AND T AND WITH D = 1 IN.

Run No.	Grading	Air Entrained?	w/c	C bags/cu yd	Slump, in.		Flow, %	
25	C	No	0.56	6.0	2.5 2.5	2.50	80.8 72.5	76.6
42	O	No	0.56	5.97	1.5 1.75	1.63	76.3 73.8	75.1
45	T	No	0.56	6.00	2.0 2.0	2.0	82.1 79.6	80.9
26	C	Yes	0.56	5.90	3.0 3.75	3.38	96.2 97.1	96.6
47	O	Yes	0.56	5.95	2.5 3.0	2.75	100.4 98.3	99.3
46	T	Yes	0.56	5.98	2.0 2.75	2.38	90.0 81.7	85.9
28	C	No	0.74	4.42	4.0 3.75	3.88	102.9 99.6	101.2
44	O	No	0.74	4.43	2.5 2.75	2.63	88.5 90.4	89.5
43	T	No	0.74	4.54	3.75 3.50	3.63	97.5 93.8	95.7
52	C	Yes	0.74	4.46	4.25 4.0	4.12	126.4 104.6	115.5
53	O	Yes	0.74	4.36	2.5 3.0	2.75	122.1 108.8	115.5
54	T	Yes	0.74	4.45	5.25 4.75	5.0	147.9 142.9	145.4

crete, which was then compacted by a vibrator. The total time of vibration was usually 15 seconds. After compaction the upper half of the concrete was removed and the coarse aggregate contents of both the upper and bottom parts were determined by washing through a No. 4 sieve. The results are presented in the columns of "Segregation %: Top-Bottom" in Tables 9 - 10. The increase of the coarse aggregate content in the bottom part which is characteristic of the segregation was expressed by the "Segregation Factor." This is defined as: The ratio of the coarse aggregate content of the bottom part of the concrete to that of the top part. Two-two measures were made from each mixture.

Test of Unit Weight. This test was performed according to the ASTM Specifications. A single determination was made for each mixture. The results are presented in Tables 11 - 12.

Test of Compressive Strength. Concrete cylinders 3-inch in diameter were made for the determination of compressive strength. This test was performed according to ASTM Specifications. Three-three measures were made from each mixture for testing the 7-day compressive strength. In addition, cylinders were made from several mixtures to test the 28-day strength. The results are presented in Tables 11 - 12.

Additional Tests. For an estimation of the unbiased value of the true experimental error, including the batch to batch variation, two mixtures were repeated and tested three times with the above mentioned tests. (Table 13.)

TABLE 9

TEST RESULTS OF BLEEDING AND SEGREGATION TESTS OF CONCRETES MADE WITH C, O, AND T GRADING AND D = 3/4 IN.

Run No.	Grading	Bleeding, cm^3 20min.	Bleeding, cm^3 100 min.	Coarse Aggregate, % Top		Coarse Aggregate, % Bottom		Segregation Factor	
13	C	5	27	31.5 32.5	32.0	35.9 37.0	36.5	1.14 1.13	1.14
15	O	40	157	39.5 38.3	38.9	43.2 42.8	43.0	1.09 1.12	1.10
16	T	19	111	27.8 31.4	29.6	37.3 34.7	36.0	1.34 1.10	1.22
14	C	10	65	29.4 32.8	31.1	38.8 39.4	39.1	1.32 1.20	1.26
18	O	10	74	38.1 39.2	38.7	43.6 45.7	44.7	1.14 1.17	1.15
17	T	8	54	25.9 30.7	28.3	38.0 37.2	37.6	1.47 1.21	1.33
19	C	7	51	30.4 29.5	30.0	38.5 38.1	38.3	1.27 1.29	1.28
21	O	14	62	36.5 36.5	36.5	41.7 43.3	42.5	1.14 1.19	1.17
20	T	8	70	24.1 25.2	24.7	36.4 35.9	36.1	1.51 1.42	1.46
23	C	19	85	28.1 28.4	28.2	39.8 39.3	39.6	1.42 1.38	1.40
22	O	15	75	30.1 32.3	31.2	45.5 45.0	45.2	1.51 1.39	1.45
24	T	15	79	28.2 25.6	26.9	35.4 34.2	34.8	1.25 1.33	1.29

TABLE 10

TEST RESULTS OF BLEEDING AND SEGREGATION TESTS OF CONCRETES MADE WITH C, O, AND T GRADINGS AND D = 1 IN.

Run No.	Grading	Bleeding, cm^3 20min.	Bleeding, cm^3 100 min.	Coarse Aggregate, % Top		Coarse Aggregate, % Bottom		Segregation Factor	
25	C	6	39	44.2 41.8	43.0	48.2 47.6	47.9	1.09 1.14	1.12
42	O	10	65	49.2 49.3	49.2	53.1 53.8	53.4	1.08 1.09	1.09
45	T	5	24	35.0 36.7	35.9	43.6 42.8	43.2	1.24 1.16	1.20
26	C	5	35	43.2 42.7	42.9	47.2 48.4	47.8	1.09 1.14	1.12
47	O	3	32	50.9 48.7	49.8	56.2 56.8	56.5	1.10 1.16	1.13
46	T	4	23	37.8 38.0	37.9	42.4 40.1	41.3	1.12 1.05	1.09
28	C	10	66	43.4 45.7	44.5	48.0 49.4	48.7	1.10 1.08	1.09
44	O	13	80	51.2 55.8	53.6	57.8 56.3	57.1	1.13 1.01	1.07
43	T	15	74	41.4 37.6	39.5	44.9 44.0	44.4	1.08 1.17	1.12
52	C	15	73	41.1 40.6	40.9	45.4 47.4	46.4	1.10 1.17	1.13
53	O	25	135	46.0 42.3	44.1	52.3 52.0	52.2	1.14 1.23	1.18
54	T	27	139	31.3 30.2	30.7	39.0 37.4	38.2	1.24 1.24	1.24

TABLE 11

TEST RESULTS OF UNIT WEIGHT AND COMPRESSIVE STRENGTH OF CONCRETES MADE WITH C, O, AND T GRADINGS AND D = 3/4 IN.

Run No.	Grading	Unit Weight lb/cu ft.	Compressive Strength, psi 7-day		28-day	
13	C	150.7	3,270 3,260 3,300	3,277	4,680 4,550 4,600	4,610
15	O	148.0	2,700 2,840 2,720	2,753	-----	-----
16	T	149.5	2,920 2,930 2,925	2,925	4,115 4,250 4,090	4,152
14	C	145.8	2,400 2,440 2,550	2,463	3,280 3,980 3,830	3,697
18	O	143.0	2,175 2,245 2,225	2,215	3,140 3,175 3,210	3,178
17	T	145.0	2,515 2,260 2,360	2,378	2,900 3,040 2,775	2,905
19	C	148.5	4,110 4,060 3,710	3,960	6,040 5,640 -----	5,840
21	O	150.0	3,860 4,010 3,610	3,830	5,875 5,990 5,675	5,850
20	T	148.7	3,920 3,840 3,810	3,860	6,110 5,575 6,010	5,898
23	C	141.3	2,830 2,730 2,840	2,800	5,090 5,050 4,390	4,850
22	O	144.3	2,760 2,880 2,620	2,750	4,790 4,980 4,835	4,870
24	T	144.8	2,975 2,875 2,835	2,895	4,895 4,950 4,980	4,940

TABLE 12

TEST RESULTS OF UNIT WEIGHT AND COMPRESSIVE STRENGTH OF CONCRETES MADE WITH C, O, AND T GRADINGS AND D = 1 IN.

Run No.	Grading	Unit Weight lb/ cu ft.	Compressive Strength, psi 7-day		28-day	
25	C	153.3	5,445 5,400 5,760	5,535	6,915 7,095 6,995	7,000
42	O	151.8	4,220 4,350 4,285	4,285	4,915 6,610 -----	5,762
45	T	152.8	5,420 5,320 5,370	5,370	6,220 6,430 -----	6,325
26	C	150.8	4,540 4,695 4,800	4,678	6,100 6,540 6,770	6,460
47	O	151.0	4,060 4,085 4,600	4,235	5,720 5,300 5,630	5,550
46	T	152.0	4,410 4,350 4,370	4,377	6,000 5,860 5,460	5,773
28	C	149.9	2,990 2,800 2,895	2,895	-----	-----
44	O	150.0	1,570 1,215 1,470	1,418	-----	-----
43	T	150.0	2,610 2,280 1,910	2,273	-----	-----
52	C	152.5	2,880 2,500 2,660	2,680	-----	-----
53	O	149.2	2,240 2,290 2,380	2,303	-----	-----
54	T	152.0	2,680 2,650 2,540	2,623	-----	-----

TABLE 13

RESULTS OF REPETITIONS OF MIXTURES NO. 13 AND 19

Run No.	Slump, in		Flow, %		Segregation Factor		7-day Strength psi	
13	1.25 1.50	1.38	94.3 87.0	89.6	1.14 1.13	1.14	3,270 3,260 3,300	3,277
56	3.75 3.75	3.75	109.2 104.6	106.9	1.10 1.03	1.06	2,705 2,740 2,745	2,733
57	2.75 2.50	2.62	106.3 100.0	103.2	1.00 1.04	1.02	2,540 2,670 2,680	2,630
19	6.25 5.25	5.75	141.2 128.8	135.0	1.27 1.29	1.28	4,110 4,060 3,710	3,960
60	8.25 7.25	7.75	158.3 161.7	160.0	1.17 1.20	1.18	3,500 3,580 3,210	3,430
62	6.0 4.4	5.20	138.0 140.0	139.0	1.23 1.00	1.12	3,660 3,440 3,280	3,460

EVALUATION OF THE TEST RESULTS

General Considerations

In the statistical evaluation the hypothesis was tested that continuous and gap gradings having the same D, m, and s parameters are equivalent with respect to concrete making properties. This hypothesis will be called the "equivalency hypothesis." To test the equivalency hypothesis the method of analysis of variance was applied to the results of determinations of slump, flow, segregation, and compressive strength. No statistical analysis was performed for the tests of bleeding and unit weight because only a single determination was made with each mixture.

The experimental layout is given in Table 6. From this it was established that the term confounded with the block is

$$I = CDW$$

since the treatment combinations fulfill the following equation (33):

$$c + d + w = 0 \quad (\text{mod. } 2) \qquad 14.)$$

Therefore the aliases are as follows:

$$\begin{aligned} D &= CW \\ C &= DW \\ W &= CD \\ A &= ACDW \end{aligned} \qquad 15.)$$

It can be seen that the factor G, which was the main subject of interest in the investigation, is not affected by confounding.

The complete model of the experimental design is the following (34):

$$y_{ijklmn} = \mu + A + D + AD + C + AC + W + AW + G + AG + DG + CG + GW + ADG + ACG + AWG + E \qquad 16.)$$

where

y one of the tested concrete properties,

D effect of the maximum particle-size on y,

A effect of the air content of concrete on y,

C effect of the cement content of concrete on y,

G effect of the grading on y,

W effect of the consistency on y, and

E error term.

In the statistical tests of the equivalency hypothesis all the main effects and all the 2-term interactions of two degrees of freedom were analyzed.

The SS and MS terms for the main effects and interactions were calculated in the usual way (35). For a test

where two determinations were performed from each batch, the term of SS (Grading) with two degrees of freedom was calculated by the following formula:

$$SS(\text{Grading}) = \frac{T_c^2+T_o^2+T_t^2}{16} - \frac{(GT)^2}{48} \qquad 17.)$$

The error term was calculated from the test results of the repeated mixtures also in the usual way. If two determinations were performed from each batch, then the error term with four degrees of freedom is the following:

$$SS(\text{error}) = \frac{\sum_1^3 (T_{1i}^2)(\sum_1^3 T_{1i})^2}{2} - \frac{}{6} + \frac{\sum_1^3 (T_{2i}^2)(\sum_1^3 T_{2i})^2}{2} - \frac{}{6} \qquad 18.)$$

The weakness of this analysis is that only 2 x 3 = 6 batches are used to estimate the total experimental error. So, the degree of freedom of the error term is 4, and therefore the power of the analysis is not very high. The adequacy of the power will be checked later.

To test the hypothesis, the F-test was used. By this test both main effects and interactions can be checked. In this experimental series the most important was the G main effect, i.e., to test the hypothesis if the C, O, and T gradings are equivalent.

As the previous formulas show, to test the significance of a main effect, the total of the similar test results were compared. For instance, in the statistical test of the G main effect on slump, the total of slumps of concretes made with C grading type was compared with the totals of slumps

of concretes made with O, and T gradings, respectively. The advantage of this method is that 24 test results are used in the comparison rather than three and therefore the conclusion obtained is more reliable.

This test method is always valid for the averages of C, O, and T gradings. However, the obtained conclusions can be applied also for the individual members of these group averages only if interaction is not present between the G main effect and the other effects. Thus, the presence of all two-factor interactions with two degrees of freedom was tested as well as the G and other main effects.

Slump Test

Analysis of Variance

The analysis of variance was performed for the values of slump test presented in Tables 7-8.

Calculation of the Error Term. The slump values of the repeated mixtures are listed in Table 13.

$$SS(Error) = 5.65 + 6.91 = 12.56$$

$$MS(Error) = \frac{12.56}{4} = 3.14$$

Calculation of the Other SS Terms. The Correction Factor (CF) is the following:

$$CF = 647.0$$

Factor G :

$T_C = 63.00$	$T_C^2 = 3,969.0$
$T_O = 45.00$	$T_O^2 = 2,025.0$
$T_T = 68.25$	$T_T^2 = 4,658.1$
Total : 176.25	10,652.1

$$SS(G) = \frac{10,652.1}{16} - CF = 18.6$$

Similarly :

$$SS(A) = \frac{15,876.5}{24} - CF = 14.4$$

$$SS(C) = \frac{15,917}{24} - CF = 16.2$$

$$SS(D) = \frac{15,974.5}{24} - CF = 18.6$$

$$SS(W) = \frac{17,170.8}{24} - CF = 68.5$$

Factor AG (Table 14) :

$$SS(Table) = \frac{5,442}{8} - CF = 33.2$$

$$SS(AG) = SS(Table) - (SS(A) + SS(G)) =$$

$$= 33.2 - 33.0 = 0.2$$

Similarly :

$$SS(GC) = 36.3 - 34.8 = 1.5$$

$$SS(GD) = 41.0 - 37.2 = 3.8$$

$$SS(GW) = 88.5 - 87.1 = 1.4$$

TABLE 14

DATA FOR CALCULATING THE AG TERM

A	G	C	O	T
Non Air Entrained		2.75	2.50	5.50
		11.50	7.00	13.25
		5.00	3.25	4.00
		7.75	5.25	7.25
		27.00	18.00	30.00
Air Entrained		7.25	3.50	8.75
		13.75	12.50	14.75
		6.75	5.50	4.75
		8.25	5.50	10.00
		36.00	27.00	38.25

TABLE 15

ANOV TABLE FOR SLUMP TEST

Source	d.f.	SS	MS	F
G	2	18.6	9.3	2.96
A	1	14.4	14.4	4.58
C	1	16.2	16.2	5.16
D	1	18.6	18.6	5.93
W	1	68.5	68.5	21.8*
AG	2	0.2	0.1	0.03
G	2	1.5	0.8	0.26
DG	2	3.8	1.9	0.61
WG	2	1.4	0.7	0.22
Error	4	12.6	3.14	

$F_{.05}$ (1,4) = 7.709

$F_{.05}$ (2,4) = 6.944

*Significant at α = 0.05 level.

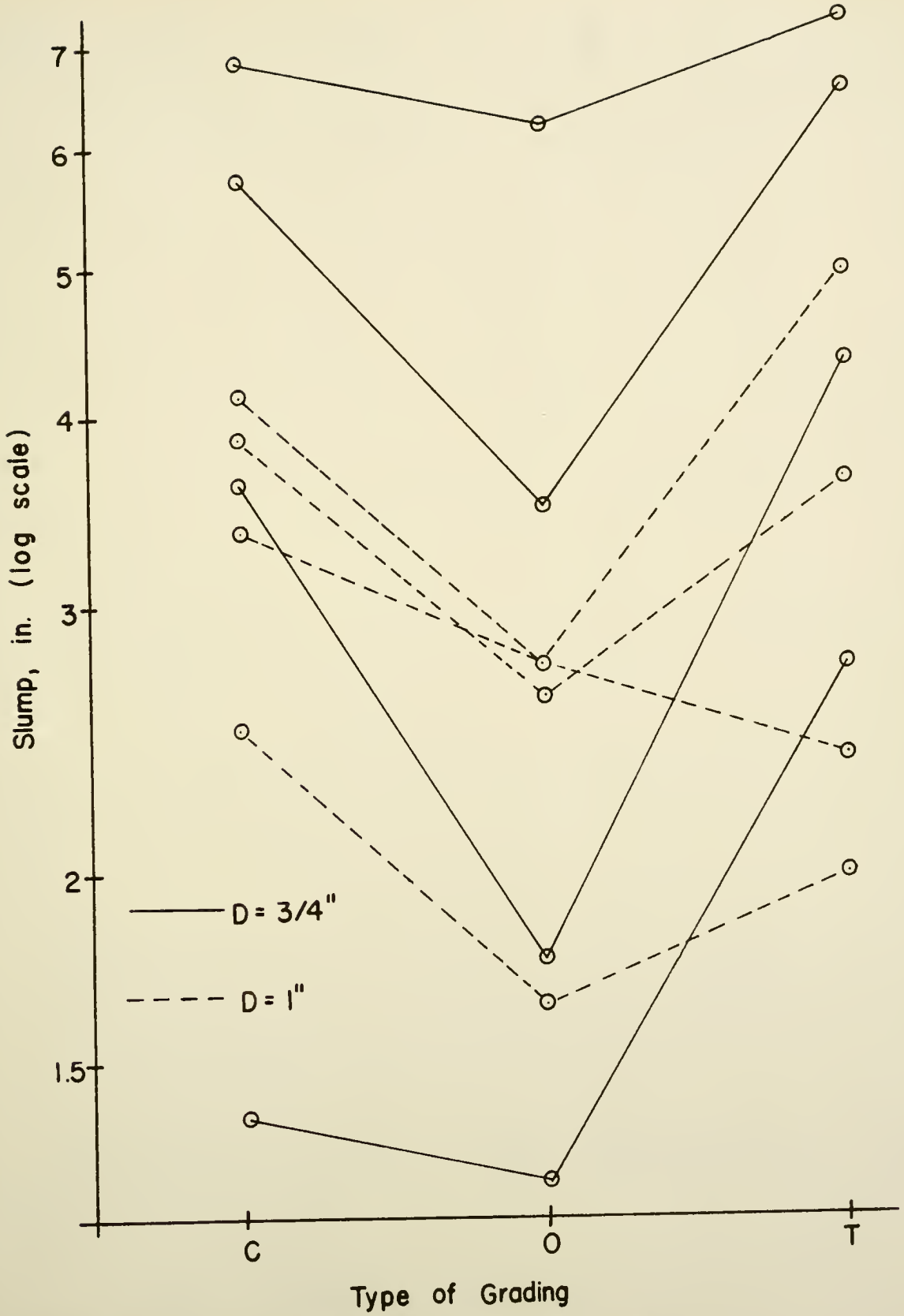


Figure 15. Slump Curves as Functions of Mix Design and Type of Grading

F-Test. The data for F-tests are summarized in Table 15.

Evaluation of the Analysis. The "Grading" main effect and all the two-factor interactions with two degrees of freedom are not significant at the $\alpha = 0.05$ level.

The presence of interactions can also be checked graphically. Such a check is presented in Figure 15. Slump curves of concretes having identical mix design are given as functions of the type of grading. Since these slump curves, with one exception, exhibit the same, nearly parallel, V shape, there is no reason to assume interactions with the G main effect.

Thus, one fails to reject the equivalency hypothesis with respect to slump at the $\alpha = 0.05$ level.

Flow Test

The results of the flow tests are summarized in Tables 7 - 8.

Analysis of Variance

Calculation of the Error Term. The flow values of the repeated mixtures are summarized in Table 13.

$$SS(\text{error}) = 290 + 721 = 1011$$

$$MS(\text{error}) = \frac{1011}{4} = 253$$

Calculation of the Other SS Terms. The correction factor is the following:

$$CF = 592,342$$

$$SS(G) = \frac{9,495,600}{16} - CF = 1133$$

$$SS(A) = \frac{14,318,300}{24} - CF = 4260$$

$$SS(C) = \frac{14,223,600}{24} - CF = 308$$

$$SS(D) = \frac{14,410,480}{24} - CF = 8100$$

$$SS(W) = \frac{14,478,420}{24} - CF = 10930$$

$$SS(AG) = 5520 - 5390 = 130$$

$$SS(GC) = 2550 - 1410 = 1109$$

$$SS(GD) = 9480 - 9230 = 250$$

$$SS(GW) = 12110 - 12060 = 50$$

F-test. The data for F-tests are summarized in Table 16.

TABLE 16

ANOV TABLE FOR FLOW TEST

Source	d.f.	SS	MS	F
G	2	1133	566	2.24
A	1	4260	4260	16.80*
C	1	308	308	1.22
D	1	8100	8100	32.00*
W	1	10930	10930	39.50*
AG	2	130	65	0.26
CG	2	1109	555	2.19
DG	2	250	125	0.49
WG	2	50	25	0.10
Error	4	1011	253	

$F_{.05}(1,4) = 7.709$

$F_{.05}(2,4) = 6.944$

*Significant at the $\alpha = 0.05$ level.

Evaluation of the Analysis. The "Grading" main effect and all the two-factor interactions with two degrees of freedom are not significant at the $\alpha = 0.05$ level. Thus, one fails to reject the equivalency hypothesis also with respect to flow at the $\alpha = 0.05$ level.

Segregation Test

The results of the segregation tests are summarized in Tables 9 - 10. The statistical evaluation was performed on the segregation factors.

Analysis of Variance

Calculation of the Error Term. The segregation factors obtained from the repeated mixtures are summarized in Table 13.

$$SS(\text{error}) = 0.014 + 0.028 = 0.042$$

$$MS(\text{error}) = \frac{0.042}{4} = 0.011$$

Calculation of the Other SS Terms. The correction factor is the following:

$$CF = 69.312$$

$$SS(G) = \frac{1109.805}{16} - CF = 0.051$$

$$SS(A) = \frac{1664.554}{24} - CF = 0.044$$

$$SS(C) = \frac{1664.528}{24} - CF = 0.043$$

$$SS(D) = \frac{1669.203}{24} - CF = 0.238$$

$$SS(W) = \frac{1665.296}{24} - CF = 0.075$$

$SS(AG) = 0.134 - 0.095 = 0.039$

$SS(CG) = 0.100 - 0.094 = 0.006$

$SS(DG) = 0.304 - 0.289 = 0.015$

$SS(GW) = 0.128 - 0.126 = 0.002$

F-test. The data for the F-test are summarized in Table 17.

Evaluation of the Analysis. The "Grading" main effect and all the two-factor interactions with two degrees of freedom are not significant at the $\alpha = 0.05$ level. Thus, one fails to reject the equivalency hypothesis also with respect to segregation at the $\alpha = 0.05$ level.

Compressive Strength

The results of the compressive strength tests are summarized in Tables 11 - 12. The statistical evaluation was performed for the 7-day strength.

TABLE 17

ANOV TABLE FOR SEGREGATION TEST

Source	d.f.	SS	MS	F
G	2	0.051	0.025	2.27
A	1	0.044	0.044	4.00
C	1	0.043	0.043	3.91
D	1	0.238	0.238	21.60*
W	1	0.075	0.075	6.81
AG	2	0.039	0.020	1.82
CG	2	0.006	0.003	0.27
DG	2	0.015	0.008	0.73
WG	2	0.002	0.001	0.09
Error	4	0.042	0.011	

$F_{.05}(1,4) = 7.709$

$F_{.05}(2,4) = 6.944$

*Significant at $\alpha = 0.05$ level.

Analysis of Variance

Calculation of the Error Term. The compressive strengths obtained from the repeated mixtures are summarized in Table 13.

$$SS(\text{error}) = 727{,}020 + 531{,}800 = 1{,}258{,}820$$

$$MS(\text{error}) = \frac{1{,}258{,}820}{4} = 314{,}710$$

Calculation of the Other SS Terms. The correction factor is the following:

$$CF = 775{,}851{,}700$$

$$SS(G) = \frac{18{,}713{,}440{,}000}{24} - CF = 3{,}874{,}900$$

$$SS(A) = \frac{28{,}090{,}150{,}000}{36} - CF = 4{,}430{,}200$$

$$SS(C) = \frac{29{,}451{,}900{,}000}{36} - CF = 42{,}257{,}200$$

$$SS(D) = \frac{28{,}125{,}298{,}000}{36} - CF = 5{,}406{,}600$$

$$SS(W) = \frac{28{,}401{,}300{,}000}{36} - CF = 13{,}073{,}200$$

$$SS(AG) = 9{,}290{,}000 - 8{,}305{,}100 = 984{,}900$$

$$SS(CG) = 46{,}264{,}400 - 46{,}132{,}100 = 132{,}300$$

$$SS(DG) = 10{,}610{,}000 - 9{,}281{,}500 = 1{,}328{,}500$$

$$SS(GW) = 16{,}973{,}300 - 16{,}948{,}100 = 25{,}200$$

F-test. The data for the F-test are summarized in Table 18.

Evaluation of the Analysis. The "Grading" main effect and all the two-factor interactions with two degrees of freedom are not significant at the $\alpha = 0.05$ level. Thus, one fails to reject the equivalency hypothesis also with respect

TABLE 18

ANOV TABLE FOR COMPRESSIVE STRENGTH

Source	d.f.	SS	MS	F
G	2	3874900	1937450	6.15
A	1	4430200	4430200	14.10*
C	1	42257200	42257200	134.3 *
D	1	5406600	5406600	17.2 *
W	1	13073200	13073200	41.6 *
AG	2	984900	492450	1.6
CG	2	132300	66150	0.2
DG	2	1328500	664250	2.1
WG	2	25200	12600	0.04
Error	4	1258820	314710	

$F_{.05}$ (1, 4) = 7.709

$F_{.05}$ (2, 4) = 6.944

* Significant at $\alpha = 0.05$ level.

to the values of 7-day compressive strength at the $\alpha = 0.05$ level.

An analysis of variance was not carried out for the results of 28-day strength because too much data were missing. It is seen from Tables 11 - 12, however, that the 28-day strengths are nearly the same within a mix design, and so there is no reason to assume that the statistical analysis of the 28-day strengths would give different results from that of the 7-day strengths. This opinion is supported by the fact that there is a significant linear correlations between the presented 7-day strengths and those of 28-day ($r = 0.94$).

Bleeding

The results of the bleeding tests are summarized in Tables 9 - 10.

It was not possible to make a statistical evaluation for the results of the bleeding tests because only a single determination was made from each mixture, for experimental reasons. A comparison of the bleeding results of concretes made with C, O, and T gradings within a given mix design shows that the differences are mostly small. Where the differences are large, they are due to factors other than the grading. For instance, the only difference in composition of mixtures No. 13 and No. 14, both made with the same continuous grading, is that Darex was added to the latter. Still, there is a more than 100% increase in the bleeding. It is

obvious that this large increase cannot be accounted for simply by the addition of Darex. There exist other factors that have a significant effect on the bleeding but that are out of control at present. Such factors can be the particle shape, the surface texture of particles, etc.

Thus, in an intuitive, semi-quantitative sense one fails to reject the equivalency hypothesis with respect to the bleeding.

Unit Weight

The test results for unit weight are summarized in Tables 11 - 12.

It was not possible to make a statistical evaluation of the results of unit weight tests because only a single determination was made from each mixture, for experimental reasons. It is seen, however, that the differences in the unit weights of concretes made with C, O, and T types of grading are usually less than 1 per cent within each group. Even the maximum difference is less than 2.5 percent.

Thus, there seems to be little reason to reject the equivalency hypothesis also with respect to unit weight. This fact is important because, when using a standardized testing and compacting method, the unit weight gives indirect information about the workability of concrete.

Check of the Power of the Analysis

In the presented analyses of variance the degree of

freedom of the error term was four. This means, that the power of the analysis is not high. The question, therefore, arises of whether the power is adequate or not. In other words, can the analyses detect differences of magnitudes that are significant with respect to concrete technology? The statistical control of this problem would be difficult in the case given because such data are needed as the true variances of the laboratory tests. These data are not available even in the literature.

Fortunately, this problem can be approached from the viewpoint of concrete technology. The technical literature includes many discussions of the effects of the factors of air content, cement content, maximum particle size, and workability on the properties of concrete (36). Thus, it is known that in general concretes of higher cement-content have higher compressive strength, the wetter concretes can segregate more easily, and the addition of air entraining admixture results in higher slump and flow values, but in lower strengths.

The presented analyses of variance are powerful enough to show all of these effects, as can be seen in the Tables 15 - 18. Thus, the power of the analyses of variance seems to be adequate.

Individual Comparisons

Although the presented analysis of variance supports the equivalency hypothesis, additional individual statistical

comparisons were made within each mix design for the various laboratory tests to check the equivalency of the C, O, and T gradings.

For these individual comparisons Tukey's studentized range test (37) was used. According to this, $\bar{Y}_i$ and $\bar{Y}_j$ are significantly different at a level α if

$$|Y_i - Y_j| > q_\alpha s_{\bar{y}} \qquad 19.)$$

where

q_α the upper 100α % point of the studentized range, for the proper combination of k and d.f.,

k total number of means, $\bar{Y}_i$, among which contrasts are to be made,

d.f. degrees of freedom on which s^2 is based,

s^2 unbiased estimate of error variance,

$s_{\bar{y}} = \sqrt{\frac{s^2}{n_i}}$ estimated standard error of $\bar{Y}_i$, and

n_i number of observations on which $\bar{Y}_i$ is based.

In the presented experiments:

$k = 3$ $df = 4$ $\alpha = 0.05$ and so $q_\alpha = 5.00$

The other values, and the L lower limits of the significant differences are summarized in Table 19.

The test results show that the differences among the results obtained with C, O, and T gradings within a mix design are always smaller than the critical L values presented in Table 19. Thus, one fails to reject the hypothesis that the

C, O, and T gradings are equivalent at $\alpha = 0.05$ also on this basis of individual comparisons.

TABLE 19

INDIVIDUAL COMPARISONS

Test	n_i	s^2	$s_{\bar{y}}$	L
Slump	2	0.0579	0.17	0.85
Flow	2	253	11.28	56.4
Segregation	2	0.011	0.075	0.375
Compressive strength	3	314,710	324	1620

Technical Considerations

The previously presented statistical evaluations, both the analysis of variance and the individual comparisons, showed that there is no reason to reject the equivalency hypothesis. If, however, one wants to check the validity of the equivalency hypothesis without statistics, by only a numerical comparison of the test results, he can find data which are doubtful support of the hypothesis. Such data are, e.g., the slump of the mix No. 18 (Table 7), or the segregation factor of the mix No. 21 (Table 9). Therefore it is desirable to check, without statistics and simply with technical considerations, how many instances of such "doubtful" data exist in the presented test series.

For this purpose, tolerance limits were established for each laboratory test for the deviations from the group average. These limits were based on the presented tests and previous laboratory experiences (38) (39). Table 20 contains the group averages, and the maximum deviations of the repeated mixtures presented in Table 13 as well as the established tolerance limits for each test. The next step was to see in how many cases the deviations of the concretes made with the gradings C, O, and T exceed these tolerance limits. For this purpose, Table 21 was prepared, which contains for each laboratory test the group averages of the test results of the concretes made with C, O, and T gradings within a mix design, and also the maximum deviations from this average. This table shows that there are only seven of more than one hundred test results for which the deviations are larger than the tolerance limits and, as such, should be considered "doubtful" with respect to the equivalency hypothesis. These values are the following:

Slumps of the mixes No. 18 and 21 (Table 7),
Flows of the mixes No. 16 and 54 (Tables 7 and 8),
Segregation factor of the mix No. 21 (Table 9), and
7-day compressive strength of the mixes No. 42 and 44 (Table 12).

It is important to note here that these larger deviations are not always disadvantageous in a technical respect. Sometimes the reason for these deviations is that one or the other

TABLE 20

AVERAGE RESULTS AND MAXIMUM DEVIATIONS OF THE REPEATED MIXTURES NO. 13 AND 19

Run No.	Slump in.		Flow %		Segr. fact.		7-day compr. str. psi.	
13	2.58	+1.17 -1.20	99.9	+7.0 -10.3	1.07	+.07 -.05	2880	+397 -250
19	6.23	+1.52 -1.03	144.7	+15.3 - 9.7	1.19	+.09 -.07	3617	+343 -187
Tolerance limits	Ave. ± 1.2		Ave. ± 15		Ave. ± 0.1		Ave. ± 0.1(Ave)	

gap grading exhibited a better technical property than the continuous grading did. For instance, the deviation of the segregation factor of the mix No. 21 (with one gap) is fairly large because this value is smaller than that of the mix No. 19 with continuous grading. A smaller segregation factor means less segregation, i.e. a larger deviation in this direction represents a larger safety. Similar cases are the flow values of the mixtures No. 16 and 24 with two-gap grading. Here the large deviations were caused by the flow values being better than those of the concretes with continuous gradings.

Thus, three of the seven presented "excessive" deviations actually increase the safety. This gives further sup-

TABLE 21

AVERAGE TEST RESULTS AND MAXIMUM DEVIATIONS OF CONCRETES MADE WITH GRADINGS C, O, AND T

Max. part. size	Air Entrained	w/c	Cem. Cont.	Slump in		Flow %		Segr. fact.		7-day str. psi	
3/4"	No	0.80	4.5	1.8	+1.0 -0.5	*98	+19 -10	1.15	+.07 -.05	2980	+290 -230
	Yes	0.80	4.5	*3.25	+1.1 -1.5	114	+12 -15	1.25	+.08 -.10	2350	+110 -140
	No	0.67	5.8	*5.3	+1.3 -1.8	135	+ 8 - 8	*1.30	+ .15 - .13	3880	+ 80 - 50
	Yes	0.67	5.8	6.8	+0.5 -0.6	149	+ 1 - 1	1.38	+ .07 - .09	2820	+ 80 - 60
1"	No	0.56	6.0	2.0	+0.5 -0.4	78	+ 3 - 2	1.14	+ .06 - .05	*5070	+470 -770
	Yes	0.56	6.0	2.8	+0.5 -0.5	94	+ 5 - 8	1.11	+0.02 -0.02	4430	+250 -200
	No	0.74	4.4	3.4	+0.5 -0.8	96	+ 6 - 6	1.09	+ .03 - .02	*2200	+700 -780
	Yes	0.74	4.4	4.0	+1.0 -1.2	*126	+20 -10	1.18	+ .06 - .05	2540	+140 -230

* One term of this average is "doubtful".

port for the practical application of the equivalency hypothesis.

On the other hand, this means that, although the presented statistical evaluations and technical considerations make the equivalency hypothesis acceptable, nevertheless not all of the presented data obviously support this hypothesis. There are a few exceptions. A qualitative explanation of this fact is presented in the next chapter. More precisely quantitative limitations of the hypothesis wait on further laboratory work.

Complementary Comments

The analysis of results of the presented experiments shows that there is no reason to reject the equivalency hypothesis of the tested gradings.

In other words, the hypothesis is acceptable that the gradings having the same D, m, and s parameters are equivalent with respect to concrete-making properties. The term "practically" indicates that not a complete identity is expected from all of these gradings but rather a close similarity.

There are two main reasons why complete identity is not to be expected among these gradings: Three parameters can characterize a particle size distribution fairly well, but not completely. Thus, different particle size distributions having the same D, m, and s parameters, are expected to have similar, but, in general, not exactly the same concrete mak-

ing properties. By increasing the number of parameters, the completeness of the characterization would improve at the expense of simplicity. However, the presented experiments show that such improvement is not worthwhile because the similarity of gradings having the same D, m, and s values is close enough for field or even laboratory purposes.

The differences among the concrete making properties of the gradings having the same D, m, and s parameters can increase if the particle shape is not the same for each fraction in the aggregate, e.g., if the fine aggregate is sand and the coarse aggregate is crushed stone. In this case the various particle size distributions usually have various crushed stone content. With the change of the quantity of crushed stone, the concrete making properties of even the same grading change. For example, it is well known that the higher the crushed stone content, the poorer the workability of the concrete, other factors being the same (40). Among the tested gradings, the type O had the highest crushed stone content, and the type T the lowest one. This fact explains why the T gradings resulted usually in the highest slump and flow values, and the O gradings in the smallest ones. Nevertheless, the actual differences among the concrete making properties of C, O, and T gradings are not excessive, as has been shown.

GRADING CORRECTION AND OTHER APPLICATIONS

Correction of Grading

The grading is evaluated by the fineness modulus method by itself in two steps.

a) The fineness modulus of the aggregate in question is determined;

b) This fineness modulus is compared with the required value.

If the difference between these two values does not exceed the permissible tolerance, the grading is adequate. However, if the fineness modulus is considerably higher or lower than the required one, the grading is inadequate. In this case the aggregate may be used for the intended purpose if its grading is corrected by proportioning two or more aggregates to produce a grading which satisfies the requirements. The necessary mix proportions are obtained in case of two aggregates from the following simultaneous equations:

$$p_1 m_1 + p_2 m_2 = m_o$$

$$p_1 + p_2 = 1 \qquad 20.)$$

The solutions of these are the two well-known formulas:

$$p_1 = \frac{m_o - m_1}{m_2 - m_1} \text{ and}$$

$$p_2 = 1 - p_1$$

The method is the same if the specific surface area is used instead of the fineness modulus.

If, however, the fineness modulus and the specific surface area are used simultaneously to characterize a grading, an extension of the method mentioned above becomes necessary. In this case a grading that has the required values of m_o and s_o can be attained, generally, by mixing at least three aggregates of different grading. The necessary mix proportions are obtained in the case of three aggregates from the following simultaneous equations:

$$\begin{aligned} p_1 m_1 + p_2 m_2 + p_3 m_3 &= m_o \\ p_1 s_1 + p_2 s_2 + p_3 s_3 &= s_o \\ p_1 + p_2 + p_3 &= 1 \end{aligned} \qquad 21.)$$

If the determinant formed from the coefficients of equation (21.) is not equal to zero, only a singular solution exists. This means that in the case of three aggregate fractions there is only one such mix-proportion, and consequently only one such grading, that satisfies the required conditions of m_o and s_o. If, however, the number of fractions is four or more, then from them one can produce an infinite number of various gradings whose fineness moduli and specific surface areas are equal to the required values.

The case presented in equation (21.) is the simplest one. It is more complicated to find solutions in an ana-

lytical way if, instead of fixed values of m_o and s_o, the permitted tolerance of the required grading is also taken into account. For example, this can happen in equation (21.) so that instead of the required m_o, the value of $m_o \pm \Delta m_o$, and instead of s_o, the value of $s_o \pm \Delta s_o$ are substituted. Usually an infinite number of such gradings can be produced from three fractions that fall within the given gradation limits. An additional difficulty is to find the totality of these solutions. Fortunately the whole process can be simplified with a graphical method (41).

Some Other Applications of the Numerical Characterization of Gradings

The author is of the opinion that the characterization of a grading by means of the simultaneous use of fineness modulus and specific surface area can be applied usefully not only within the sphere of concrete technology but also in other areas where the correct characterization of grading is important. As examples, some possibilities will be presented from two technical areas.

Bituminous Mixtures

In designing bituminous mixtures the specific surface area concept is frequently applied openly or tacitly. This method would be made more complete by the additional application of fineness modulus--an opinion supported by the observation that a similarity appears to exist between the

bitumen requirements of a bituminous mixture and the mixing water requirements of a concrete mixture with respect to the workability of the mixtures. In connection with the latter instance the application of the fineness modulus proved to be useful. The explanation of the expressed similarity lies perhaps in the fact that both the bitumen and part of the mixing water have a similar role in the mixtures, namely the reduction of friction among the aggregate particles (lubrication effect).

In many cases the grading of the aggregates for bituminous mixtures is specified by fraction limits and some qualitative indication of the particle size distribution as "one size aggregate", etc. Instead of this method an entirely numerical characterization would seem to be more precise; that is, the upper limit of grading can be characterized numerically by the maximum particle size; the descriptive concepts such as "fine", "coarse", etc., and "sand", "crushed stone", etc., by the maximum particle size and the fineness modulus; and the concepts such as "one size", "open grade", and "dense grade", describing the type and extent of the particle size distribution, by means of the specific surface area.

Soil Mechanics

The numerical characterization of gradings could also be used more extensively within the sphere of soil mechanics. Classification of coarse-grained soils or the coarse fraction of other soils might be made numerical instead of descriptive. The descriptive

concepts such as "sand", "gravel", etc., and "fine", "coarse", etc., could be replaced by the maximum particle size and fineness modulus, and the concepts describing the type and extent of particle size distribution (well graded, uniformly graded, etc.) by means of the specific surface area.

Thus, one could probably obtain a more complete picture of the uniformity of the grading of a soil by means of the specific surface area than by the usual "uniformity coefficient" since the numerical value of specific surface area takes into consideration the whole particle size distribution. The uniformity coefficient, however, considers only two points on the sieve curve. Burmister characterized the grading of soil mixtures by the so-called "size factor" (42) which is similar to the fineness modulus concept. According to the studies of Burmister, the borrow-pit materials of those dams in which slides occurred had a high "size factor".

The application of the fineness modulus concept to characterize the soil gradings is supported by the fact that the particle size distributions of soils approach very well the logarithmic normal distribution in many cases (43). Furthermore, it has been shown previously that the fineness modulus is proportional to the average of the logarithmic normal distribution.

SUMMARY OF RESULTS

After presentation of the advantages and disadvantages of the fineness modulus concept, this paper reports two recent mathematical forms for the fineness modulus:

$$m = 0.0332 \left[100 \log 10D - 0.4343 \int_{0.1}^{D} \frac{f(d)}{d} \, dd \right]$$

and

$$30.1m = \int_{0.1}^{D} (\log 10d) \, f'(d) dd = \text{ave } (\log 10d)$$

where

m fineness modulus

d particle size, in mm

D maximum particle size, in mm, and

f(d) cumulative distribution function of grading.

The latter relation shows that the value of the fineness modulus is proportional to the logarithmic average of the particle sizes.

On the basis of the optimum values of the fineness modulus, an interesting relationship results between the workability and the maximum permissible values of the average particle size of a concrete mix. This is

$$\log (100 \frac{d_{ao}}{D}) = -0.263 \log D'' + 0.95$$

where

d_{ao} maximum permissible value of the average particle size of the concrete mix

D'' maximum particle size, in inches.

The second part of the thesis deals with the specific surface area concept.

The specific surface area is supplied by the following general formula:

$$s = \beta \int_{d_{min}}^{D} \frac{f'(d)\, dd}{d}$$

where

s specific surface area

d particle size

D maximum particle size

$f'(d)$ first derivative of the cumulative distribution function of grading

β constant which depends on the specific gravity, and the supposed shape of particles, units of measure, etc.

The following statement is also proved:

The specific surface area of an aggregate is proportional to the second moment of a distribution function of particle size.

Then various methods to calculate the specific surface area are presented. In these methods two assumptions are made:

a) The shape of particles is spherical;

b) The surface area of very small particles should not be taken into account with their full values.

The thesis shows that these assumptions do not hurt the general validity of the specific surface area concept.

It is also shown that there is some formal similarity between the fineness modulus and specific surface area concepts.

After the introduction of the fineness modulus and specific surface area as statistical concepts, the proposal is substantiated theoretically that the values of the maximum particle size, fineness modulus and specific surface area should be employed simultaneously in evaluating a grading.

A corollary of this proposal is the following statement: All those gradings are equivalent with respect to concrete technology that have the same maximum particle size, fineness modulus and specific surface area. This is called the "equivalency hypothesis".

This statement is logical because it says that a distribution is characterized sufficiently by its extent, average, and variance. However, it was also necessary to prove this method of evaluating grading by tests. Therefore, the

paper presents results of various concrete tests.

A significant portion of the results was analyzed by statistical methods (single and multiple correlation, analysis of variance). These analyses support the equivalency hypothesis.

CONCLUSIONS

Based on the materials studied and the tests performed, the results of this work seem to justify the following conclusions :

1. The fineness modulus of an aggregate grading is proportional to the logarithmic average of a particle size distribution.

2. The specific surface area of an aggregate grading is characteristic of the variance of a particle size distribution.

3. Aggregate gradings with the same values of the maximum particle size, the fineness modulus, and the specific surface area will result in concretes with nearly identical strengths and properties of the plastic concretes.

RECOMMENDATIONS FOR FURTHER WORK

The investigation cannot be considered accomplished on this research area even after the presented theory and experimental work. The writer thinks the following items are important for investigation in the future:

The related values of the optimum fineness modulus and specific surface area, as a function of the cement content and maximum particle size.

The numerical evaluation of the effect of particle shape and surface texture on the concrete making properties.

Exact investigation of the effect of fine particles (passing sieve No. 200) in respect of the specific surface area concept.

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